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WAVELET TRANSFORM FOR TIME-FREQUENCY ANALYSIS OF VIBRATIONAL SIGNATURE AND ITS APPLICATION

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ABSTRACT

Wavelet transform is applied to the analysis of vibration signatures in order to verify the ability of the detection of abnormal condition. It can well describe the dynamics of the signal's spectral composition of a non-stationary and stationary signal to be measured and presented in the form of 3-D time-frequency map. Although wavelet has been developed over about ten years in the mathematics and physics, its engineering applications is a first stage. The objective of this report outlines the definition of the wavelet transform and is to discuss the properties of the wavelet transform as new tool for the vibration analysis, and then demonstrates how it may be applied to the machinery condition monitoring.

I. INTRODUCTION

Wavelets are a very popular topic of the signal processing and applied mathematics. In the last ten years, an interest in them has grown at an fast rate in signal and numerical analysis [Beylkin, Coifman and Rokhlin, 1991, Heil, 1990 and Resnikoff and Burrus, 1990]. Wavelet analysis appears to new subject for the time-frequency analysis of the vibrational signatures. Traditional spectral analysis provides spectral values which are independent of time. It is assumed to be ergodic and stationary signal with

time. However the signal associated with most vibrational phenomena are in general time varying, which means that their characteristics change with time and they have various features in different time frames. For example, the vibration during the start-up of an engine or pump is non-stationary, the sound pressure generated from speaker is nonstationary, and so on. In case of the signal containing some transient or nonstationary conditions, the traditional approach in signal analysis fails to describe the dynamics of the signal's frequency components changes. Changes in the condition of a component such as a gear can be expected to cause some change in the vibration generated by mechanical system. Very little damage detection can be performed using the vibration signal directly from the system because the small changes generated by early damage may be masked by the normal vibration of the system.

Two methods of signal analysis for nonstationary application are commonly used in time-frequency domain. The windowed Fourier transform, otherwise so called short time Fourier transform(STFT), has a short time window of a fixed size centered at time t as figure 1(a). The windowed Fourier transform is given as follows

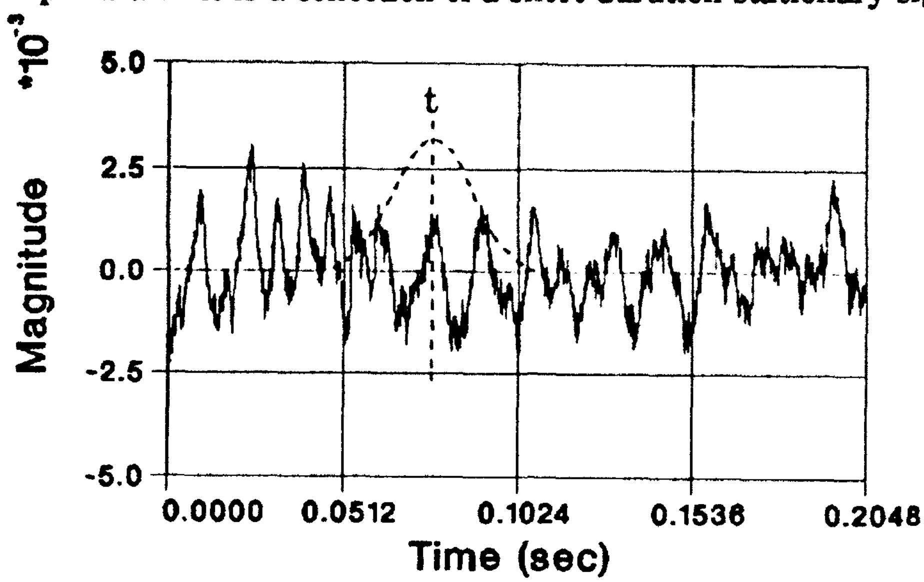
$$F(\omega,t) = \int g(\tau - t)\, e^{-i\omega\tau}\, s(\tau)\, d\tau \qquad (1)$$

where $F(\omega,t)$ is the windowed Fourier transform, $g(t)$ window function and $s(t)$ time signal. The range of integration is from $-\infty$ to ∞. If the length of the window is time duration T, the resolution of time frequency domain depends on T. Its frequency bandwidth or frequency resolution is approximately 1/T. Therefore this method have the limitation of resolution in both time and frequency domain simultaneously.

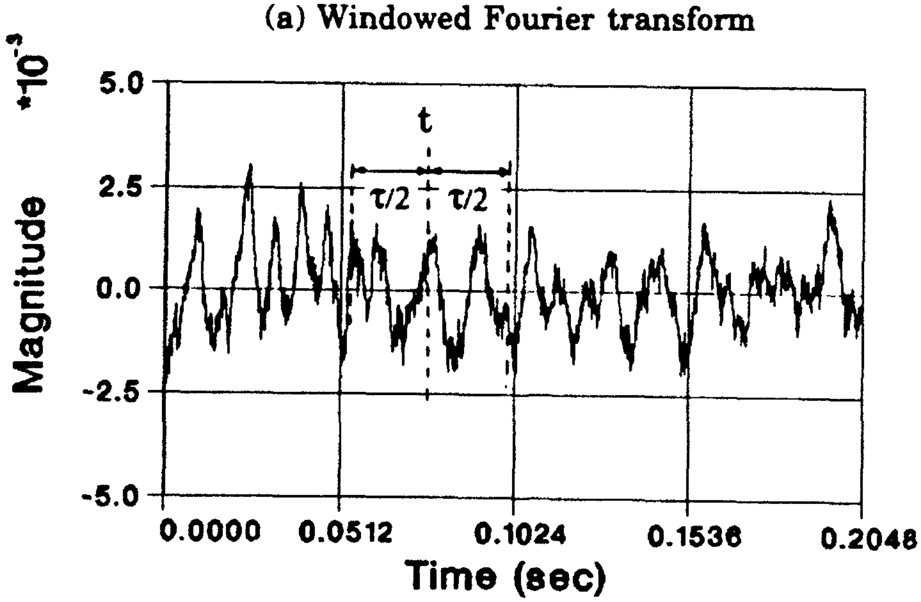
The second method, often called Wigner distribution method, is based on the instantaneous power spectra defined as following equation (2)

$$w(\omega,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} s(t-\tau/2)\, s(t+\tau/2)\, e^{-i\omega\tau}\, d\tau \qquad (2)$$

where $w(\omega,t)$ is Wigner distribution function. Equation (2) is called the Wigner distribution of $s(t)$in figure 1(b). We discussed well at references [Jeon and Shin (1993) and Shin, Jeon and Spooner (1993)] about the characteristics of Wigner-Ville distribution. For a nostationary signal analysis, spectrogram is commonly used, which is based on the assumption that it is a collection of a short duration stationary signal.



(a) Windowed Fourier transform



(b) Wigner distribution

Figure 1. Calculation of the windowed Fourier transform and instantaneous spectral density(Wigner distribution) of a vibration signal.

A major drawback of this approach is that the frequency resolution is directly affected by the duration of short stationary time, which subsequently determines the time resolution. The frequency and time resolution of the Wigner distribution are not determined by the short duration but rather determined by the selection of desired resolution of the signal itself, but may not be appropriate for signals containing patterns at both large and very small scales.

The windowed Fourier transform is well portray the characteristics of signals in which all of the patterns appear at approximately the same scale, but may not be appropriate for signals containing patterns at both large and small scales because of the time fixed window size. The multifrequency channel decomposition, which are an intermediate between a time and a Fourier representation, have found many applications in signal and image processing [Mallat, 1989]. Much recent research has been focused on this domain with the modeling of a new decomposition called the wavelet transform, which is presented signal by summation of family of functions which are the dilations and translations of a unique function called a wavelet. Instead of portraying a signal into harmonic functions ($e^{i\omega t}$ in Fourier transform), the signal is presented into a series of orthogonal basis functions of finite length. Each wavelet is located at a different position on the time axis. At the finest scale, wavelets may be very short indeed; at a coarse scale, they may be very long. Alternatively very small disturbances in a record of machinery vibration can be easily characterized from a wavelet 3-D or 2-D map in which the mean-square value of the time record is shown over wavelet scale and position.

One important property of a wavelet transform is its ability to characterize easily the local regularity of a function. Simply by a change of the scale parameter(dilation) in the wavelet transform, many scales of local structure can be described by a distribution in the time-scale plane. Wang and McFadden (1993) introduced the advantage for examining the vibration signal generated by a gear and D. E. Newland (1993) well investigated the properties of the wavelet as a new tool for the analysis of vibration records.

This report outlines the definition of the wavelet transform, compare with the advantages and the disadvantages of wavelet and pseudo Wigner-Ville distribution in time-frequency domain analysis, and then demonstrates how it may be applied to analysis of the vibration signals for the machinery condition monitoring.

II. DEFINITION OF WAVELET TRANSFORM

To overcome the limitation of the fixed resolution of the windowed Fourier transform in the time and frequency domains by decomposing the signal s(t) into a family of functions which are the translation and dilation of an unique function $\psi(t)$, defined the continuous wavelet transform as

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{\infty}^{-\infty} \overline{\psi(\frac{t - b}{a})}\, s(t)\, dt \tag{3}$$

where $W(a,b)$ is wavelet transform, ψ is an analyzing wavelet, a represents a time dilation, b a time translation, and bar for complex conjugate. The normalization factor $1/\sqrt{a}$ is perhaps most effectively visualized as endowing $|W(a,b)|^2$ with unit of power/Hertz [Shensa, 1992].

We consider the space $L^2(\mathbf{R})$ of measurable function ψ, defined on the real line $\mathbf{R}$ ($\mathbf{R} := (-\infty, \infty)$), certain weak 'admissibility' conditions are usually required on $\psi(t)$ [Shensa, 1992];

$$\int_{\infty}^{-\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|}\, d\omega < \infty \tag{4}$$

where $\hat{\psi}(\omega)$is the Fourier transform of $\psi(t)$. They ensure that the transformation is a bounded invertible operator in appropriate space [Daubechies, 1988]. If $\hat{\psi}(\omega)$ is differentiable, then it suffices that ψ be zero mean, that is,

$$\int_{\infty}^{-\infty} \psi(t)\, dt = 0 \tag{5}$$

for equation (4) to be satisfied. In particular, since the local average values of every function in $L^2(\mathbf{R})$ must 'decay' to zero at $\pm\infty$, the sinusoidal(wave) functions $e^{i\omega t}$(basis of Fourier transform) do not belong to $L^2(\mathbf{R})$. Fourier series representation of any functions is in $L^2(0,2\pi)$. In fact, if it looks for

wavelets that generate $L^2(\mathbf{R})$, these wavelets should decay to zero at $\pm\infty$; and for all practical purposes, the decay should be very fast [Chui, 1992].

In signal processing, the significance of equation (3) is well understood by comparing it to the windowed Fourier transform (or short-time Fourier transform):

$$F(\omega,b) = \int_{\infty}^{-\infty} g(t - b)\, e^{-i\omega t}\, s(t)\, dt. \tag{6}$$

Thus, to obtain $F(\omega, b)$, one multiplies the signal by an appropriate window g(t) (such as Gaussian) centered at time b and then takes the Fourier transform. In mathematical forms, equation (6) is an expansion of the signal in terms of family of functions $g(t\text{-}b)e^{i\omega t}$, which are generated from a single function g(t) through translations b in time and translations ω in frequency. In contrast, the wavelet transform of equation (3) is an expansion in function $\psi((t - b)\,/\,a)$ generated by translation b and dilation a in time. Thus, the continuous wavelet transform is similar to windowed Fourier transform with a different window size for each frequency. The important facts of this is that, while the basis functions in equation (6) have the same time and frequency resolution at all points of the transform plane, those of wavelet transform have the time resolution which decrease with increasing a and the frequency resolution which increase with decreasing a width adapted to their frequency components: at high frequency ψ are very narrow, while at low frequency ψ are much broader. As a results, the wavelet transform is better than the windowed Fourier transform to analyze on very small disturbance, i.e., high frequency phenomena. This property can be a best advantage in signal processing since high frequency characteristics are generally highly localized in time whereas slowly varying signals require good low frequency resolution [Shensa, 1992]. Figure 2 shows the typical shapes of windowed Fourier transform functions and wavelet.

The wavelet transform means that signal s(t) is characterized by decomposition into a set of wavelet family with series of different frequency

bandwidth. For the decomposition of the signal s(t) defined on real line, it is necessary to shift ψ along **R**. Let **Z** denote the set of integers;

$$\mathbf{Z} = \{ \ldots\ldots -1, 0, 1, \ldots\ldots \}.$$

The simplest way for ψ to cover all of **R** is to consider all the integral shift of ψ,

$$\psi(t - k), \qquad k \in Z. \tag{7}$$

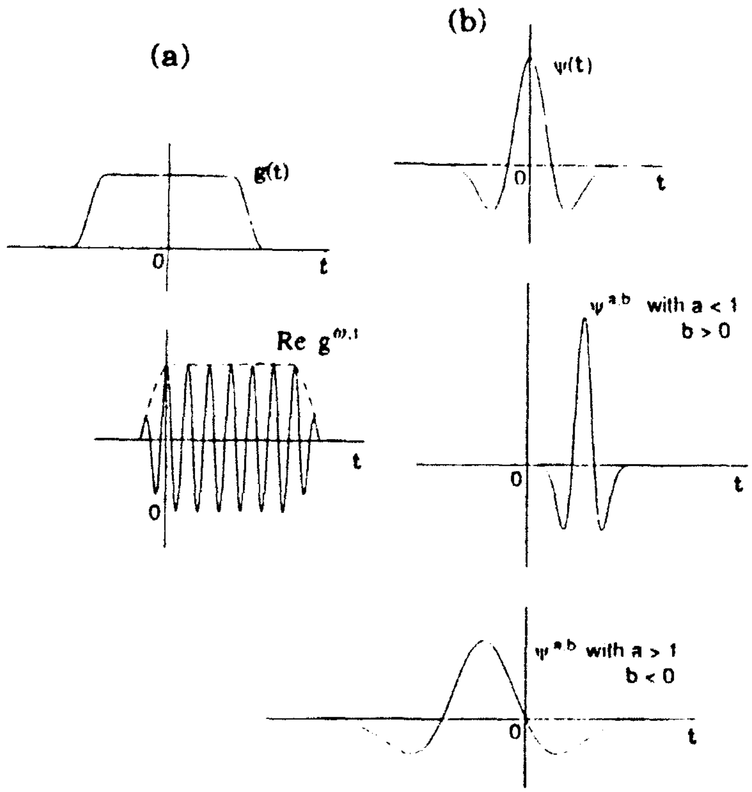


Figure 2. The typical shapes of (a) windowed Fourier transform function g(t), (b) wavelet $\psi^{a,b}$, $\psi(t) = (1 - t^2)\, e^{-t^2/2}$[Daubechies, 1992].

Next, as in Fourier transform, it must be also considered wavelets with different frequencies. It is considered wavelets with frequencies partitioned into consecutive 'octaves' (or frequency bands). For computational efficiency, it will be used integral power of 2 for frequency partitioning, that is, the small wavelets is considered as follows

$$\psi(2^j t - k), \qquad j,k \in Z. \tag{8}$$

From the equation (3), $\psi(2^j t - k)$ is obtained from the wavelet function $\psi(t)$ by dilation of $1/2^j$ and translation of $k/2^j$. An important particular case of the discrete wavelet transform which was found is that some wavelets $\psi(t)$ exist such that $\sqrt{2^j}\ \psi(2^j(t - 2^{-j}k))_{(j,k)\in Z}$ is an orthonormal basis of $L^2(\mathbf{R})$ [Mallat, 1989].

As originally proposed by Morlet et al. (1982), ψ was a modulated Gaussian

$$\psi(t) = e^{i\omega_0 t}\, e^{-t^2/2} \tag{9}$$

and this function is selected to analysis the vibration of gear box for signal processing applications[Wang and McFadden, 1993]. The example of a shape for the modulated Gaussian wavelet is shown in figure 3. The Fourier transform of the first wavelet family $\psi(t/a)$, i.e., no translation, is

$$\hat{\psi}(a\omega) = a\, e^{-(\omega - \omega_0/a)^2 a^2/2} \tag{10}$$

which has analysis frequency ω_0 / a. ω_0 / a is a simple frequency parameter which determines the analyzing wavelet. We can easily see that equation (9) satisfies the admissibility condition equation (4). The analyzing bandwidth of the wavelet is proportional to $1/a$, thus having a constant relative bandwidth(BW), that is, BW/(ω_0 / a) = constant. This feature is also reflected in the narrow time window at higher frequency(i.e., at smaller a). In general, the function $\psi(t)$ is selected by its time and frequency localization properties [Daubechies, 1990].

The scale change may be performed by substituting $a = 2^{-j}$ for the computation efficiency. In this case the wavelet family is $\sqrt{2^j}\ \psi(2^j\ t)$, and the definition of the wavelet transform becomes

$$W(2^j, b) = \frac{1}{\sqrt{2^j}} \int_{\infty}^{-\infty} \psi\left(\frac{t - b}{2^j}\right) s(t)\, dt \tag{11}$$

In this report, the unique function $\psi(t)$ can be selected to be a well-behaved modulated Gaussian function, given by equation (9).

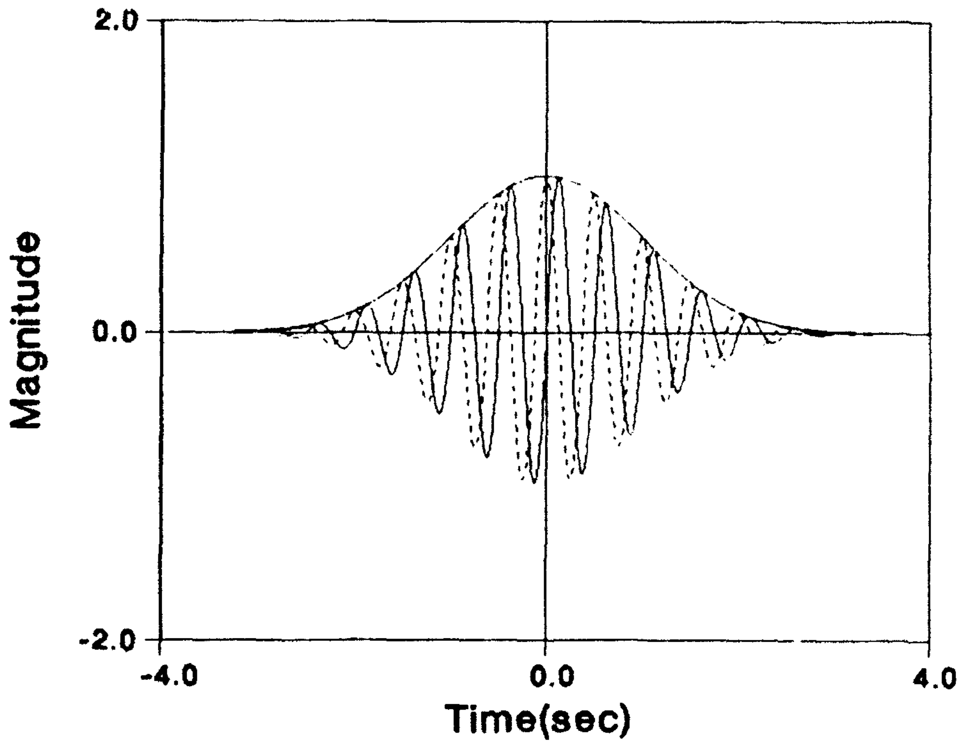


Figure 3. Example of modulated Gaussian wavelet.
$(\psi(t) = e^{i\omega_0 t} e^{-t^2/2},\ \omega_0 = 2\pi f_0,\ f_0 = 2\ \text{Hz})$

III. TIME-FREQUENCY ANALYSIS

Suppose that ψ is any basic wavelet such that both ψ and its Fourier transform $\hat{\psi}$ are window functions which have centers and half widths of time and frequency domain given by t^*, ω^*, Δt, $\Delta\omega$, respectively. Then the wavelet transform of an analog signal s(t) is given as follows.

$$W(a,b) = |a|^{-1/2} \int_{-\infty}^{\infty} s(t)\, \overline{\psi(\frac{t-b}{a})}\, dt \tag{12}$$

Equation (12) localizes the signal with a time window

$$[b + at^* - a\Delta t,\ b + at^* + a\Delta t],$$

where the center of the window is at $b + at^*$ and the width is given by $2a\Delta t$. This is called time localization in signal analysis.

On the other hand, let set the Fourier transform of ψ

$$\eta(\omega) = \hat{\psi}(\omega + \omega^*), \tag{13}$$

where η is the Fourier transform of ψ, then η is also a window function with center at ω^* and width by $\Delta\omega$, and by Parseval identity, the wavelet transform in equation (12) becomes [Chui, 1992]

$$W(a,b) = \frac{a|a|^{-1/2}}{2\pi} \int_{-\infty}^{\infty} \hat{s}(\omega)\, e^{ib\omega}\, \overline{\eta(a(\omega - \frac{\omega^*}{a})}\, d\omega \tag{14}$$

where the phase shift of $e^{ib\omega}$ is determined by translation along time axis. Equation (14) is also localized information of spectrum $\hat{s}(\omega)$ of the signal s(t) with frequency window

$$[\frac{\omega^*}{a} - \frac{1}{a}\Delta\omega, \frac{\omega^*}{a} + \frac{1}{a}\Delta\omega],$$

where the center of window is at ω^*/a and width is given by $2\Delta\omega/a$. This is called frequency localization.

From equation (12) and (14), time-frequency window of wavelet transform is as follows.

$$[b + at^* - a\Delta t, b + at^* + a\Delta t]\text{x}[\frac{\omega^*}{a} - \frac{1}{a}\Delta\omega, \frac{\omega^*}{a} + \frac{1}{a}\Delta\omega] \tag{15}$$

The covered area by time-frequency window is the multiplication of time and frequency bandwidth around the center of window. The time-frequency window is shown figure 4. Eventually it is considered positive frequencies, i.e., a > 0, the basic wavelet ψ may be chosen that the center ω^* of $\hat{\psi}$ is a positive number. The ratio of the center frequency to the width of frequency band is given by

$$\frac{\omega^*/a}{2\Delta\omega/a} = \frac{\omega^*}{2\Delta\omega} \tag{16}$$

which is independent of the location of the center frequency. This is called constant percentage band width analysis or constant-Q frequency analysis. In this report, the octave band is used for the analysis of vibration signal. The frequency window along the frequency axis narrows for large center frequency ω^*/a and widens for small one and the time window is opposite to frequency window. The area of the window is a constant, given by $4\Delta t\Delta\omega$.

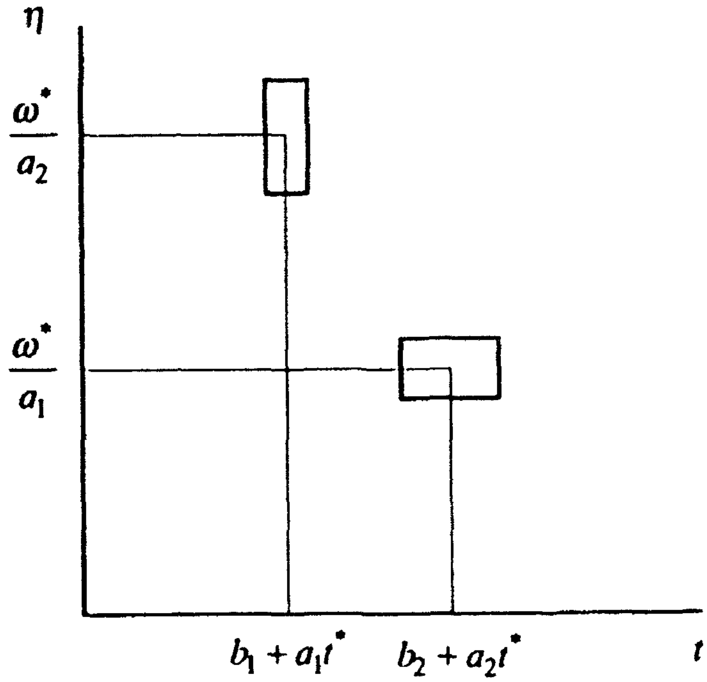


Figure 4. Time-frequency windows, $a_1 > a_2$.

IV. DISCRETE WAVELET TRANSFORM

In the continuous wavelet, the family is considered

$$\psi^{a,b}(t) = |a|^{-1/2}\ \psi(\frac{t - b}{a}) \tag{17}$$

where $b \in \mathbf{R}$, $a \in \mathbf{R}_+$ with $a \neq 0$, and ψ is admissible. **R** denotes the real line and $\mathbf{R}_+$ denotes the positive real line. It is considered with discrete values for a and b. For the discretization of the dilation parameter, we select $a = a_0^m$, where $m \in \mathbf{Z}$, and the dilation step $a_0 \neq 1$ is fixed. For convenience, it assume $a_0 > 1$. If $a_0 = 1$, the wavelet transform may be similar to windowed Fourier transform. For $m = 0$, it seems natural as well to dicretize b by taking only the integer multiples of one fixed b_0, where b_0 is appropriately chosen so that the $\psi(t - nb_0)$ cover the whole line. We arbitrarily fix $b_0 > 0$ in this report. For the different values of m, the width of $a_0^{-m/2}\ \psi(a_0^{-m}\ t)$ is a_0^m times the width of $\psi(t)$ defined at section III, so that the choice $b = nb_0a_0^m$ will ensure that the discretized wavelets at level m cover the line in the same way that $\psi(t - nb_0)$ do. Thus it is selected ,$a = a_0^m$, $b = nb_0a_0^m$, where m,n range over $\mathbf{Z}$, and $a_0 > 1$, $b_0 > 0$ are fixed; the appropriate choices for a_0 and b_0 depend on the wavelet ψ and the characteristics of signal. The discrete form of equation (17) is given as following equation [Daubechies, 1992]

$$\begin{aligned}\psi_{m,n}(t) &= a_0^{-m/2}\ \psi(\frac{t - nb_0a_0^m}{a_0^m}) \\ &= a_0^{-m/2}\ \psi(a_0^{-m}t - nb_0)\end{aligned} \tag{18}$$

For the computation efficiency, we assume that $a = 2^m$, that is, $a_0 = 2$, where m is termed the octave of the transform. This means that the

frequency resolution of wavelet has a octave band. The integral equation (3) yields a wavelet series as following equation by using equation (18).

$$W(2^m, nb_0) = \frac{1}{\sqrt{2^m}} \int_{\infty}^{-\infty} \overline{\psi\left(\frac{t - nb_0}{2^m}\right)}\, s(t)\, dt \tag{19}$$

At the discrete wavelet transform, the finite energy for the wavelet transform is not equivalent to finite energy for the wavelet series. It depends on the sampling grid as well as the function $\psi(t)$. In addition, it often take b to be a multiple of a.

$$W(2^m, n2^m) = \frac{1}{\sqrt{2^m}} \int_{\infty}^{-\infty} \overline{\psi\left(\frac{t}{2^m} - n\right)}\, s(t)\, dt \tag{20}$$

A logical step in applying the theory to discrete signal is to discretize the integral in equation (20) as follows.

$$W(2^m, n2^m) = \frac{1}{\sqrt{2^m}} \sum_k \overline{\psi\left(\frac{k}{2^m} - n\right)}\, s(k) \tag{21}$$

Octave m is only output every 2^m samples. In this form the resulting algorithm will not be translation invariant [Mallat, 1982]. The discrete wavelet transform is highly not invariant under translations. In practice one does not use an infinite number of scales, but cuts off very low and very high frequencies.

V. EXAMPLES AND DISCUSSIONS

A signature generated by machinery involves many informations about its operating condition. It can be obtained the information about the operating condition of machinery by applying the analysis tools for the vibration records. Wavelet transform is a new tool that is particularly suited for time-frequency analysis of nonstationary or stationary signals. There are many advantages of using wavelet transform for both steady and transient signals. We will discuss the performance of the wavelet transform by using simple example and compare with pseudo Wigner-Ville distribution(PWVD) in time-frequency domain analysis.

Before showing some examples, it is necessary to discuss how best to describe the results. We have found that 3-D map and 2-D map of wavelet transform are a useful presentation for many applications. The square of the amplitude in equation (21) has the unit power/Hz. The distribution of the amplitude square over the individual wavelet and position can now be seen as figure 5. If the length of sampled data is shorter than the wavelet size, the distribution at lower wavelet level, i.e., low frequency, has the value at only one position.

In order to describe the results of wavelet transform, we use the 3-D and 2-D(for the complicated signal) graphics. And for graphic, the results of that is distributed and reduced to 256(3-D) or 512(2-D) data point along the time axis. Frequency axis is a log scale (octave scale or wavelet level) and time axis is a linear scale in figures of wavelet transform.

A. Harmonic wave with stepwise frequency changes

Figure 6 shows (a) the pure sine wave with stepwise frequency changes 500 Hz, 250 Hz and 100 Hz, (b) its PWVD and (c) the wavelet transform. The wavelet transform and PWVD well represent the time delay and the

frequency components of signal. The wavelet transform is a result with frequency partitioned into consecutive octaves for computational efficiency. Then the magnitude of 100 Hz is dispersed. But the wavelet transform clearly describes the time delay. From these figures, we can see that the PWVD has the higher resolution than the wavelet transform in frequency line. However the computation time of PWVD is extremely higher than the wavelet transform. Figure 7 shows the wavelet transform of the sine wave with 500 Hz in time from 0.085 sec *to* 0.17 sec. the wavelet transform well represents the time delay and the frequency band of the signal.

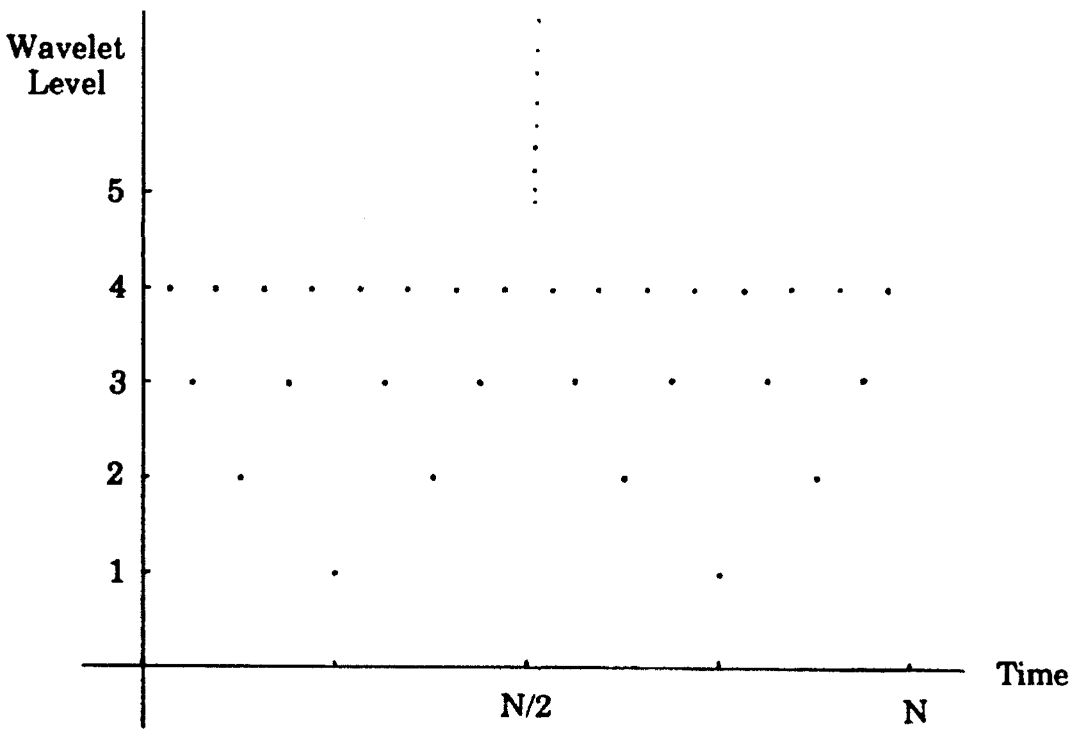
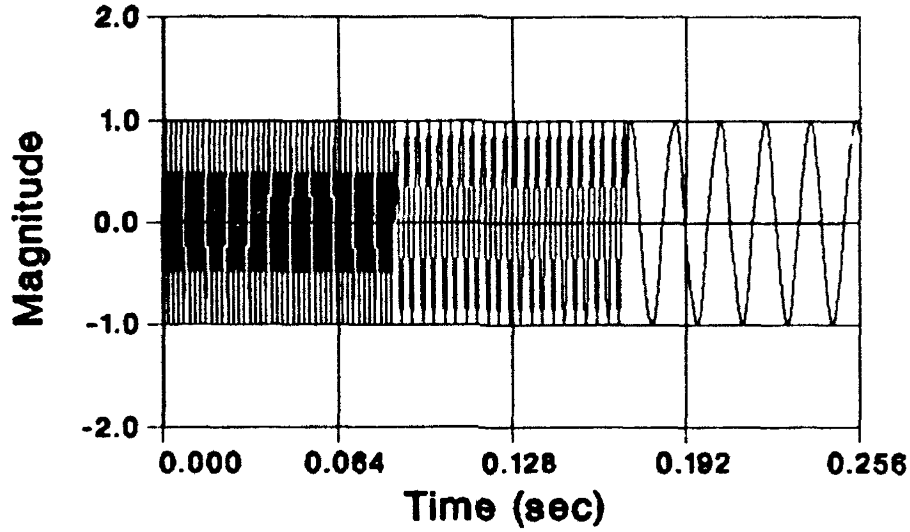
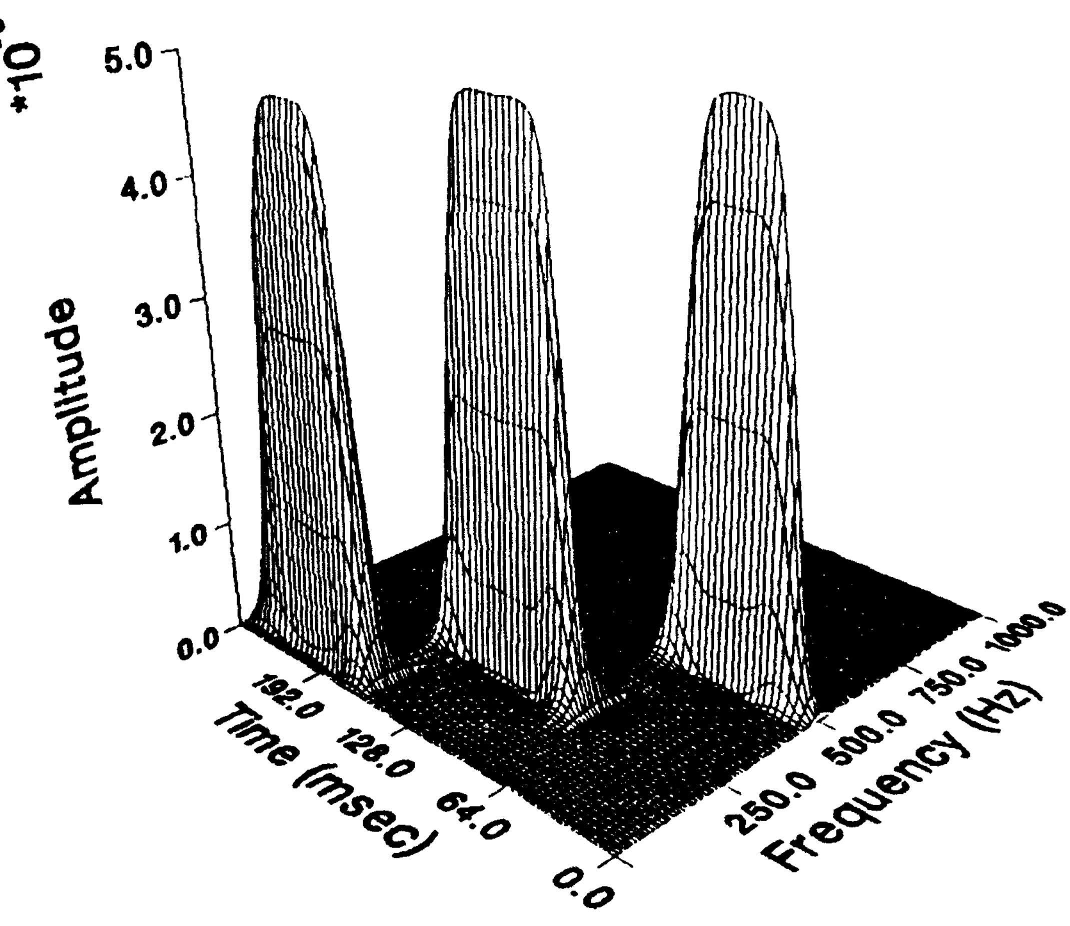


Figure 5. The lattice of time-frequency localization of wavelet transform (N = Number of data points)

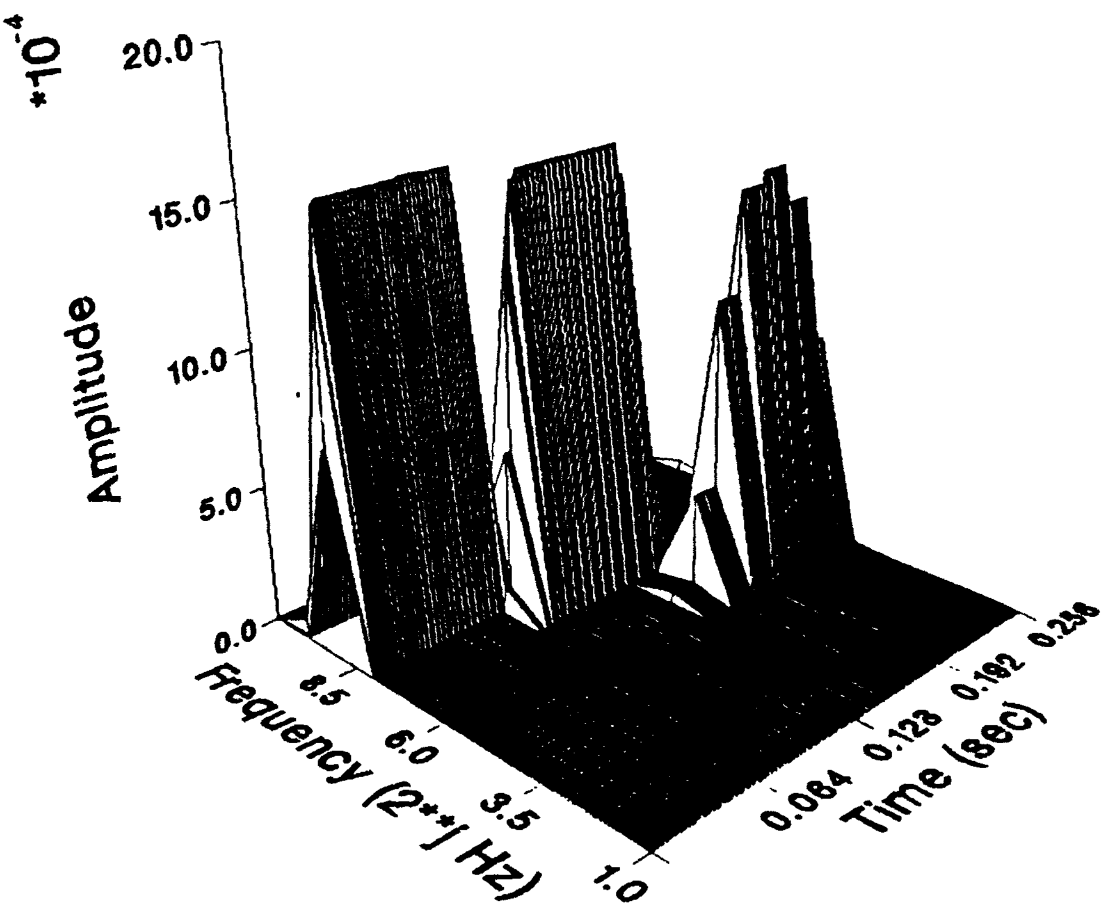


(a)



(b)

Fig.6 (continued)



(c)

Figure 6. Time-frequency localization of PWVD and wavelet transform: (a) signal s(t), (b) its PWVD and (c) its wavelet transform (f_S=2000 Hz, N=512).

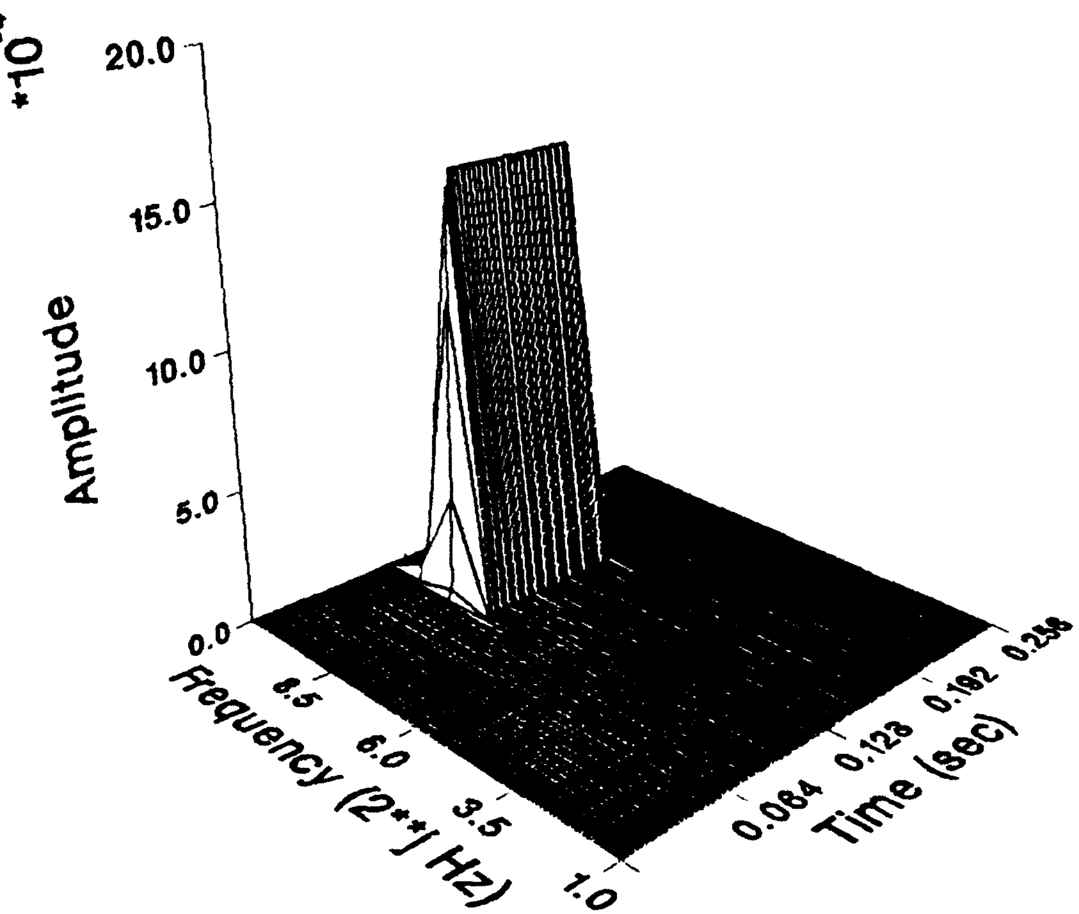
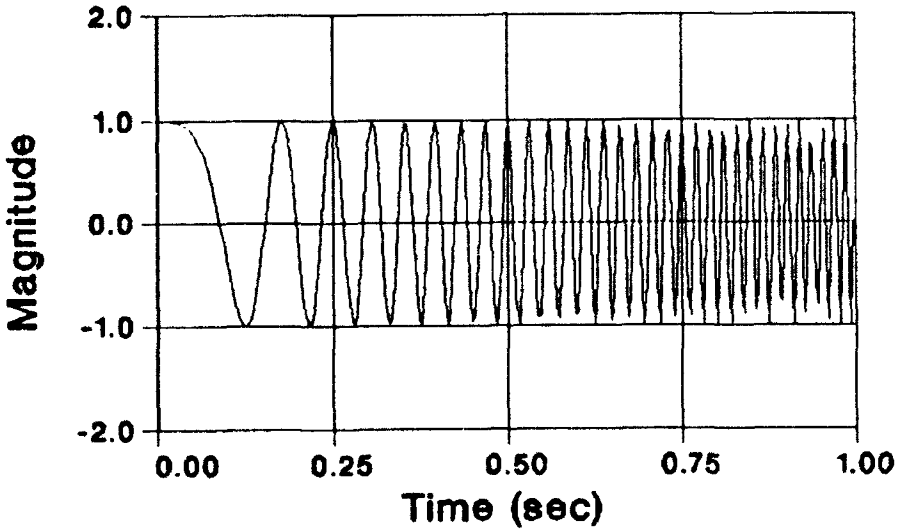


Figure 7. Wavelet transform for step sine wave (f = 500 Hz); f_s = 2000 Hz, N=512.

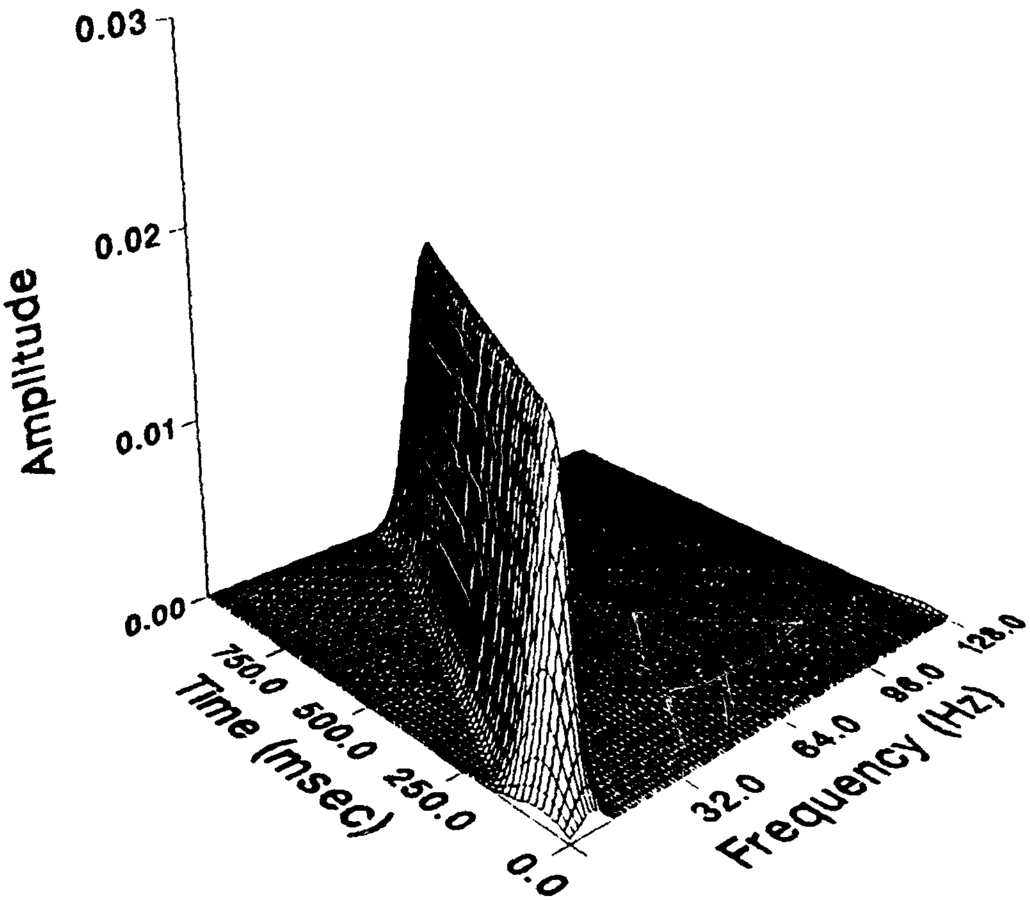
B. Swept harmonic wave

The effect of sweep rate on PWVD was well investigated at Jeon and Shin (1993), and Shin, Jeon and Spooner (1993). Figure 8. shows (a) the swept cosine wave with the sweep rate 32 Hz/sec, (b) its PWVD and (c) its wavelet transform. From this figures, we can see that PWVD is a useful tool for analysis of the signal with fast frequency change in time. However if the records of signal is longer, the computation time may be much needed. The result of the wavelet transform does not clearly represents the sweep condition but well describes the change of frequency range with time.



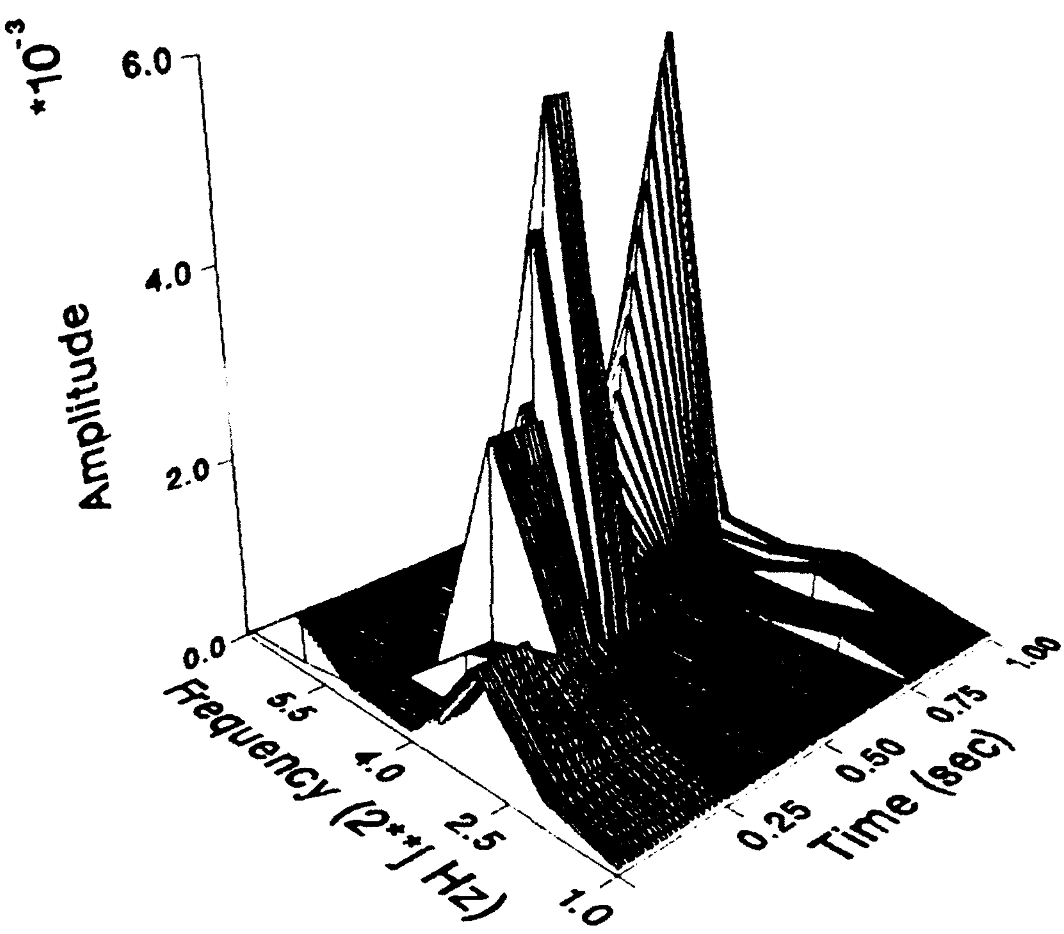
(a)

Fig.8 (continued)



(b)

Fig.8 (continued)

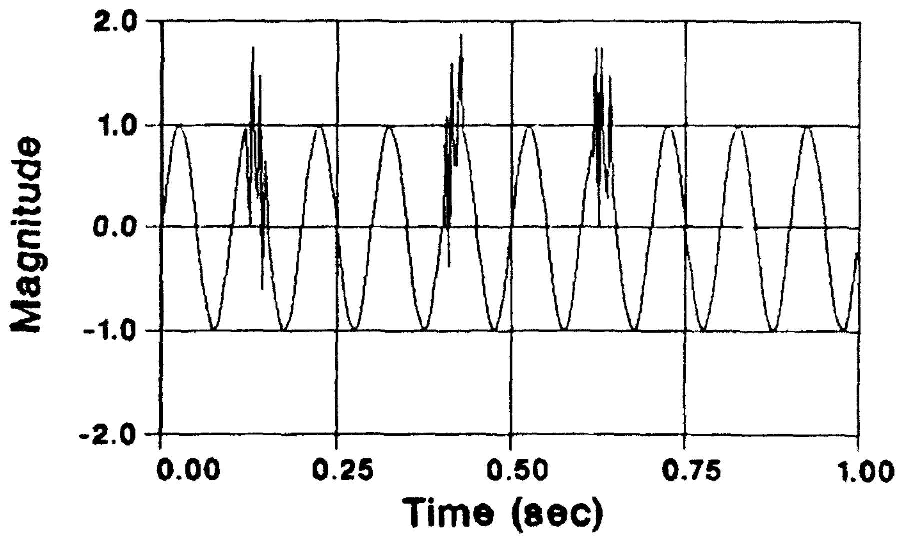


(c)

Figure 8. Time-frequency localization of the PWVD and wavelet transform: (a) signal $s(t) = \cos(2\pi\ 32\ t^2)$, (b) its PWVD and (c) its wavelet transform (f_S = 256 Hz, N = 256).

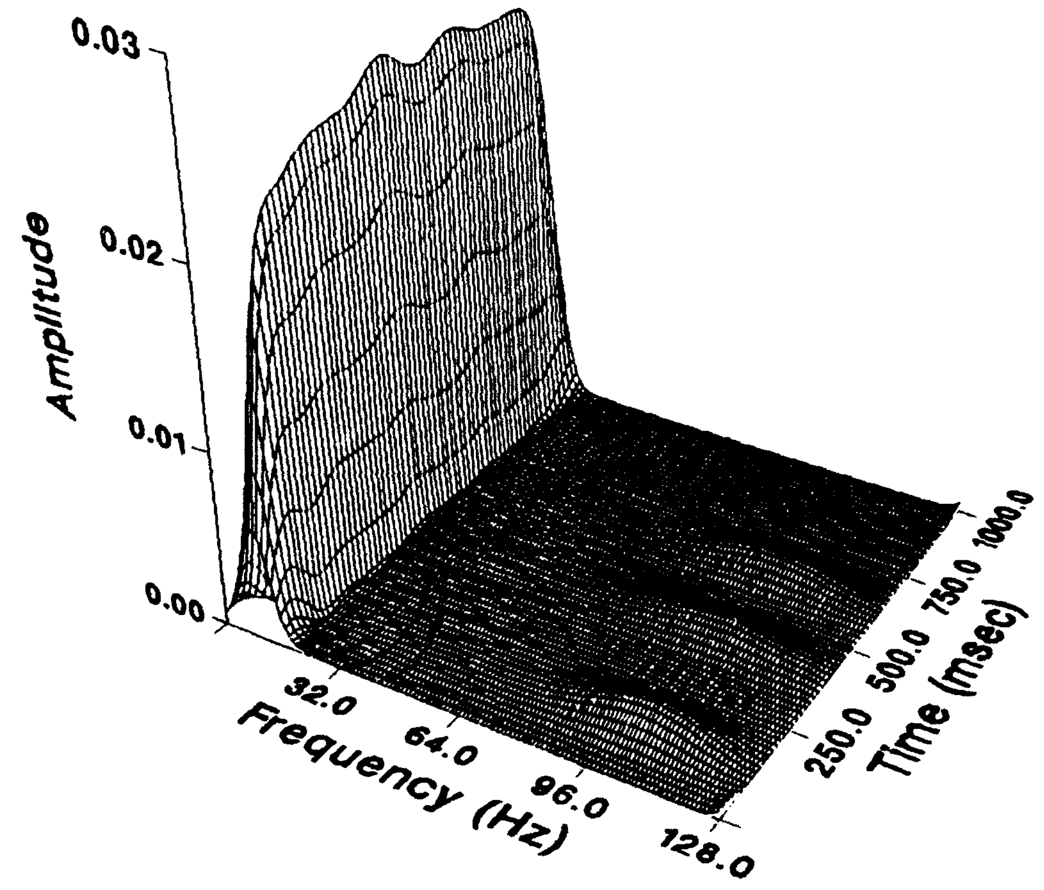
C. Harmonic wave with glitches

The interesting phenomena on the signal with an abnormal component as a fault were investigated. Figure 9 shows (a) the harmonic wave with glitch at a small region, (b) is PWVD and (c) its wavelet transform. It can be seen that both PWVD and wavelet transform of the signal figure 9(a) well represents the location of each glitch and its frequency components. Figure10 is the result of wavelet transform in case of sampling frequency 8192 Hz. From these figures, it can be clearly seen that the wavelet transform is very useful tool in the analysis of the signal required higher time resolution. Figure 9(c) shows the glitch components more clearly than PWVD because the wavelet transform has the very narrow time window in high frequency region. This characteristic of the wavelet transform is useful to detect the fault or glitch although the fault is small and to monitor the condition on any vibrational machinery under the steady operation condition. Also the wavelet transform is more effective for the analysis of signal which the time record length is long, since the sweep along the frequency line is octave step .



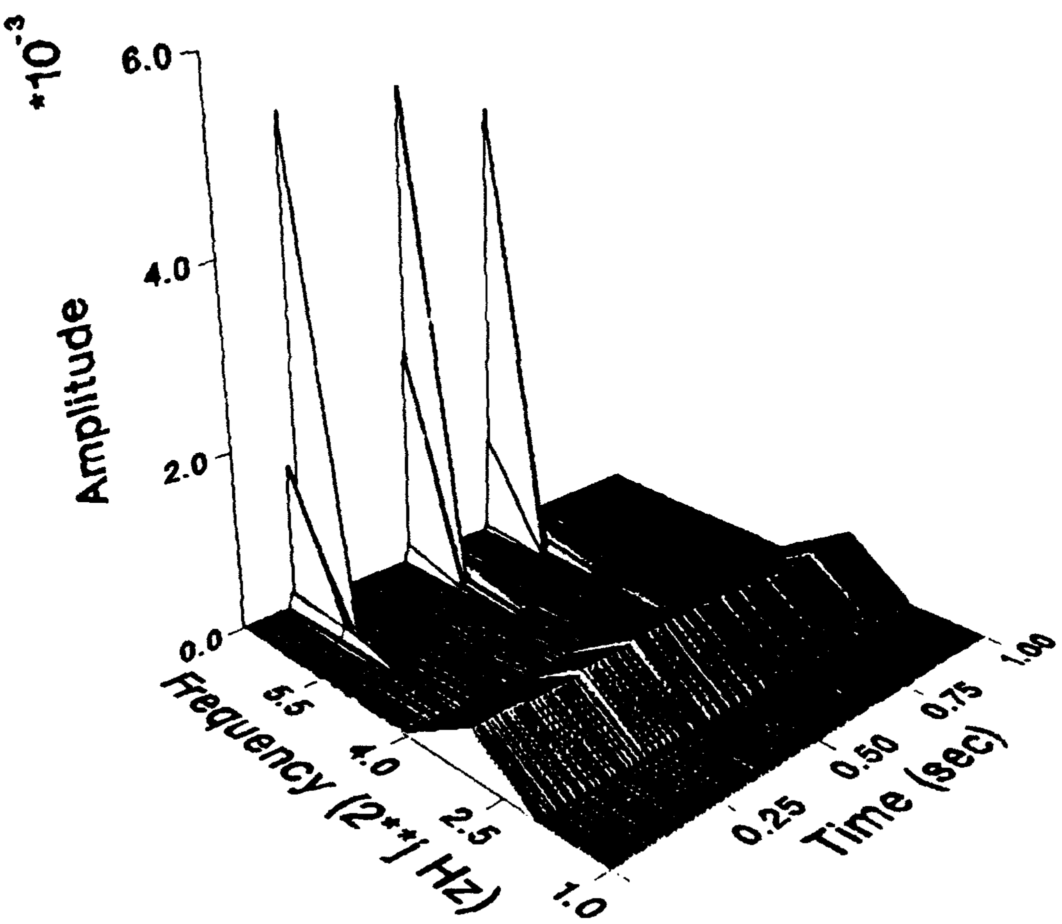
(a)

Fig.9 (continued)



(b)

Fig.9 (continued)



(c)

Figure 9. Time-frequency localization of PWVD and wavelet transform: (a) the signal s(t) [Jeon and Shin, 1993], (b) its PWVD and (c) its wavelet transform (f_S = 256 Hz, N = 256).

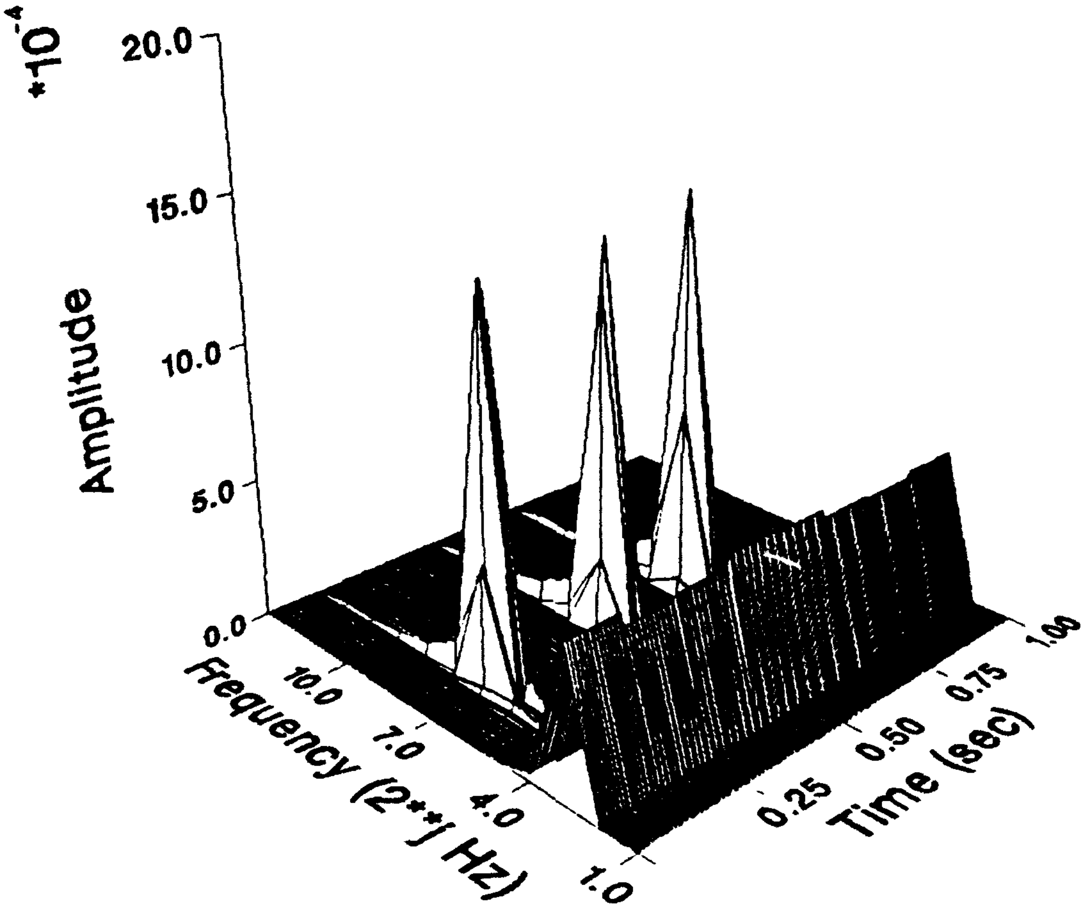


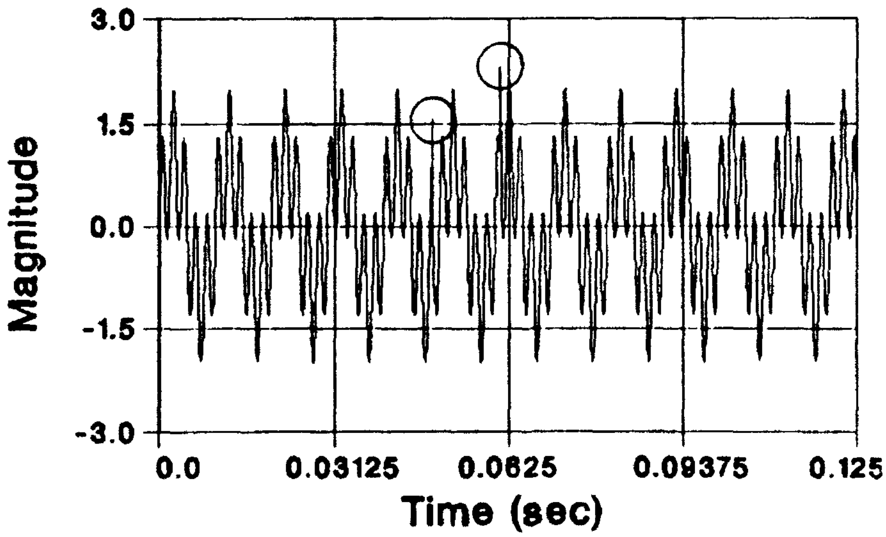
Figure 10. Time-frequency localization of wavelet transform for the signal Fig. 9(a) (f_s = 8192 Hz, N = 8192).

D. Harmonic wave with pulse

Figure 11 illustrates about the signal including small changes generated by early damage. In practice, this signal is not given by this continuous expression, but by samples, and adding a δ-function is then approximated by adding a constant to one sample only. The example signal is generated by following equation.

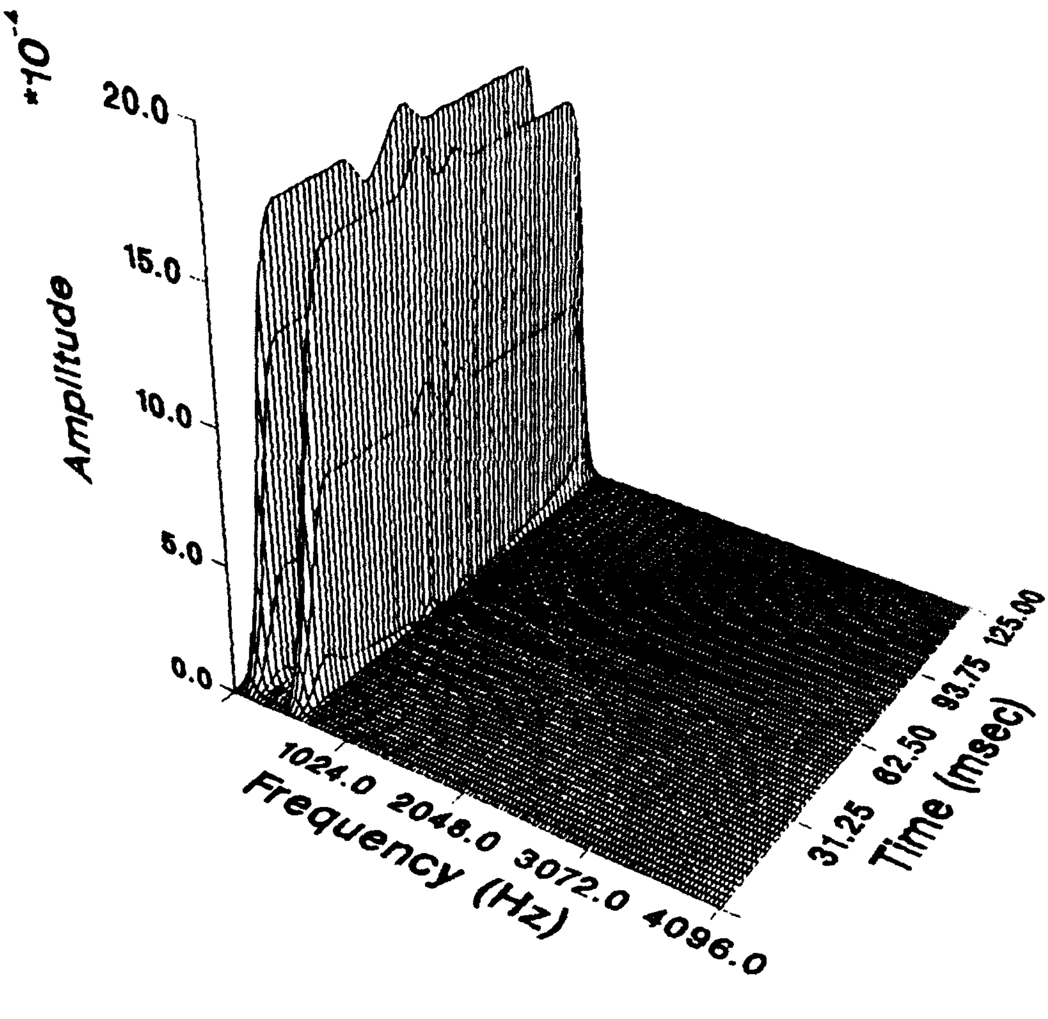
$$s(t) = \sin(2\pi\, 100\, t) + \sin(2\pi\, 500\, t) + 1.5\, \delta(t-0.049) + 1.5\, \delta(t-0.061) \qquad (22)$$

From this figure, we can see that the wavelet transform has the great advantage for the analysis of the signature including the small disturbance. PWVD does not give the exact information about time delay of pulse but well represents about the frequency components of main signatures. The wavelet transform does not describe the magnitude of the main frequency components but well represents the time delay of pulse and is obtained the result by very short computation time. From this example, the wavelet transform is very useful tool to detect the fault although that is very small region on time axis.



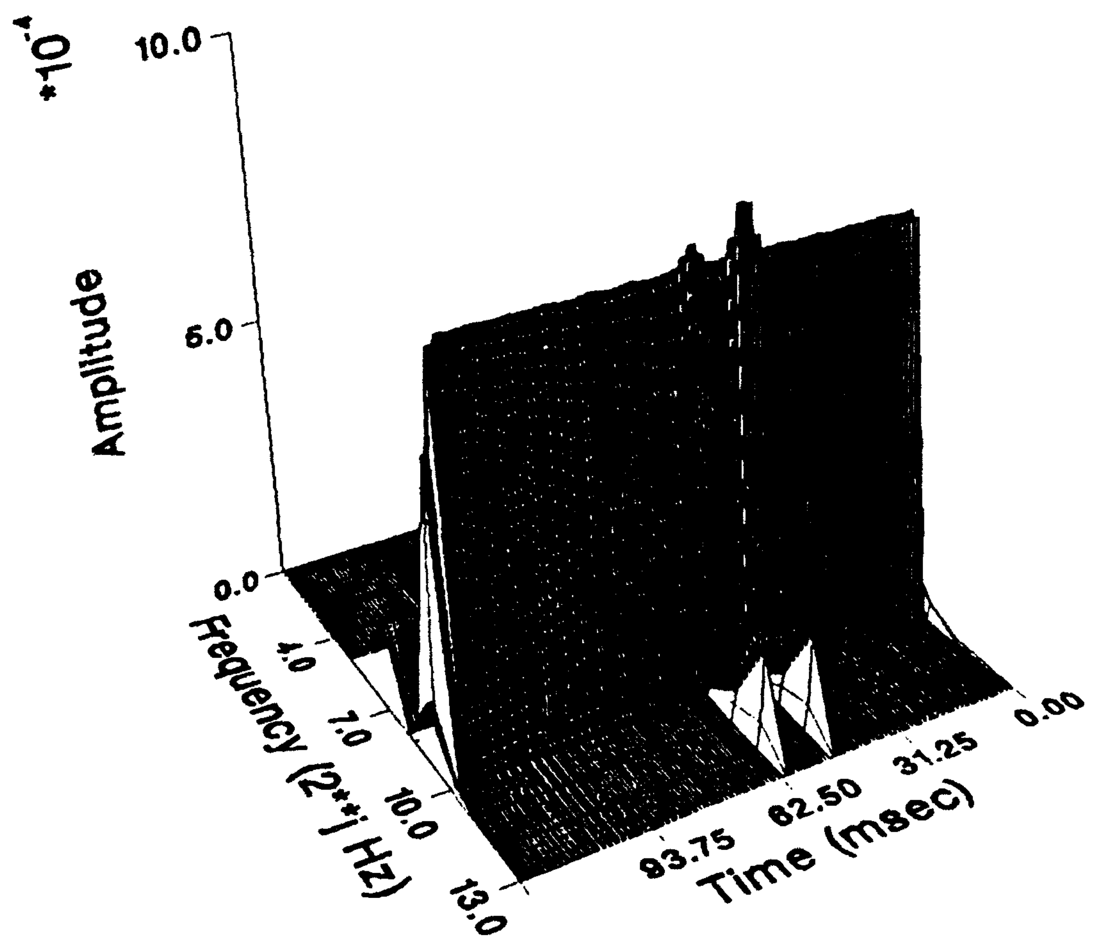
(a)

Fig.11 (continued)



(b)

Fig.11 (continued)



(c)

Figure 11. Time-frequency localization of PWVD and wavelet transform: (a) the signal s(t), (b) its PWVD and (c) its wavelet transform (f_S = 8192 Hz, N = 1024)

E. Actual pump signal

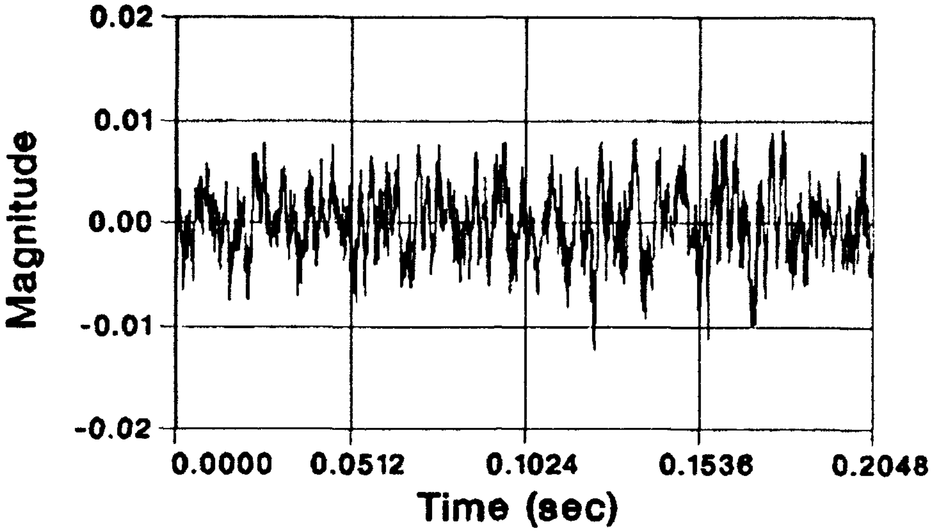
The signal of actual pump was measured at the steady state speed without and with abnormal condition. Figure 12 show the time signal pattern for each condition, and figures 13 and 14 show the results of wavelet transform by using 3-D or 2-D graphics. Also figure 15 are the results of pseudo Wigner-Ville distribution. These figures show a very interesting results.

From these figures, we can easily find the abnormal condition. At 13th frequency band of figure 13(a) and (b), the results of wavelet transform obviously show the different condition and pattern for each case. And also at 8th frequency band, the small changes of patterns can be seen. Figure 14(a) and (b) well represent the operation condition although the shape is more complicate and we can see the abnormal state of pump.

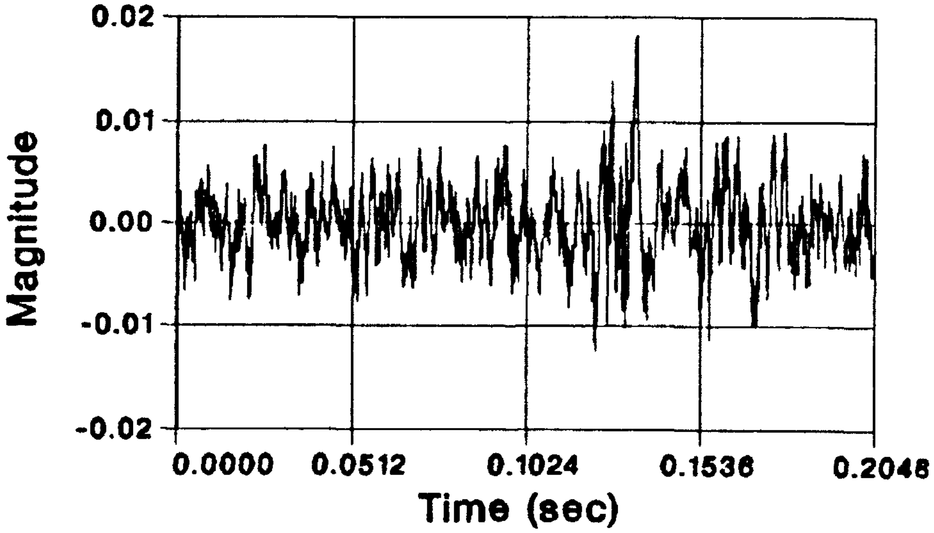
In this case, if one wavelet to have 13th frequency band is used, it gives good results for diagnosis and vibration condition monitoring as very short computation time. Figure 15 are PWVD. Also PWVD well represents the abnormal condition but the computation time is very long than that of wavelet transform. In the case of wavelet transform, it is possible to independently compute for each wavelet level different from PWVD. The difference of computation time for the wavelet transform and PWVD is about 100 times for the number of sample data N=2048 in the calculation of a whole plane. At VAX3520, the computation time is 18 seconds for wavelet transform and about 30 minutes for PWVD.

Especially the wavelet transform may be very useful to detect the small disturbance over long record length and to analyze the signals which has a long time duration and intermittent abnormal condition such as non-rotating machinery. The advantage of the Wigner-Ville distribution is that, unlike the wavelet transform or the windowed Fourier transform, it does not introduce a reference function(such as wavelet or window function in Eq.(3) and (1)) against which the signal has to be integrated and the short

time signal. The disadvantage is that the signal enters in Wigner-Ville distribution in a quadratic rather than linear way, which causes many interference phenomena as shown in references Jeon and Shin (1993), and Shin, Jeon and Spooner (1993). Wigner-Ville distribution may be useful in some application for signals which have a very short time duration.

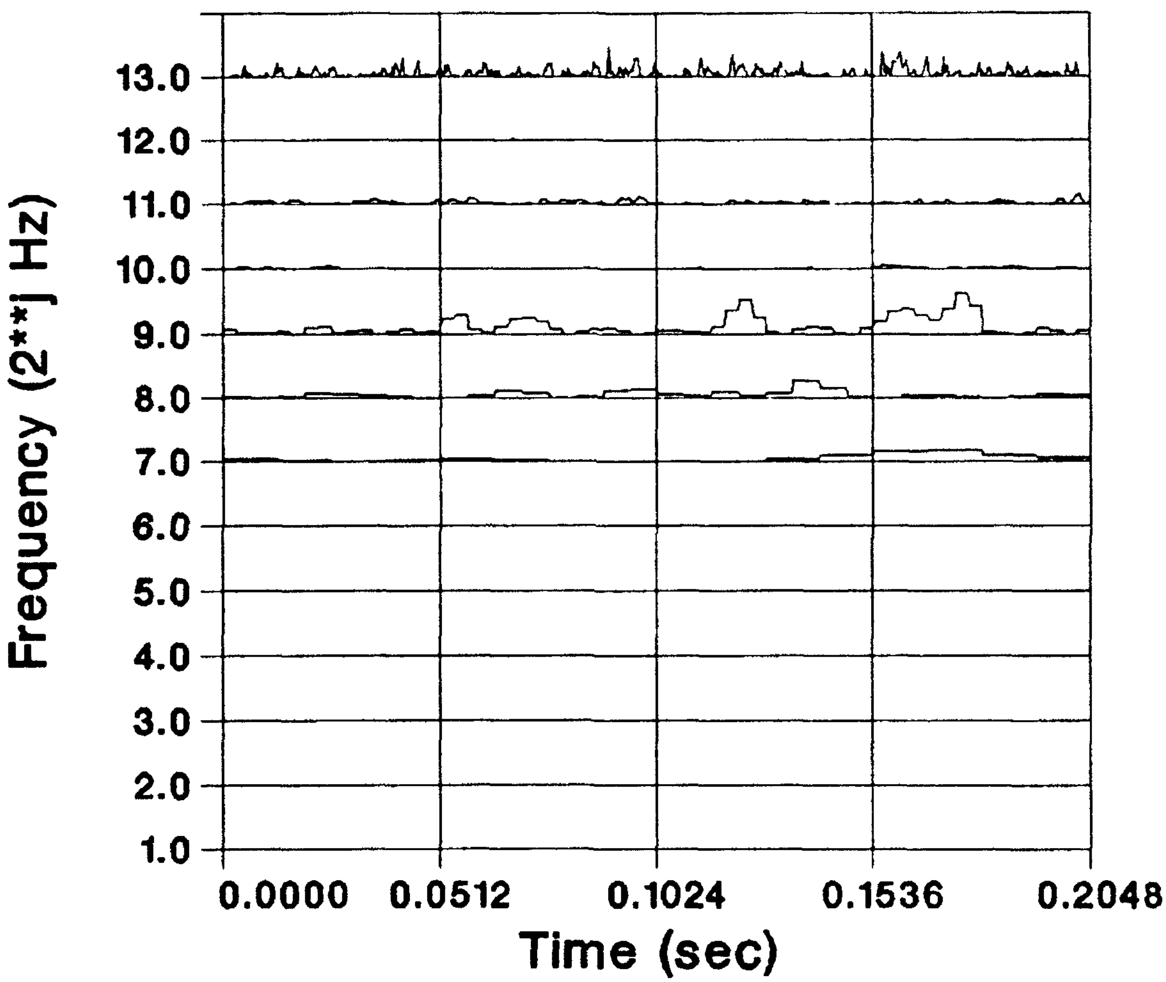


(a)



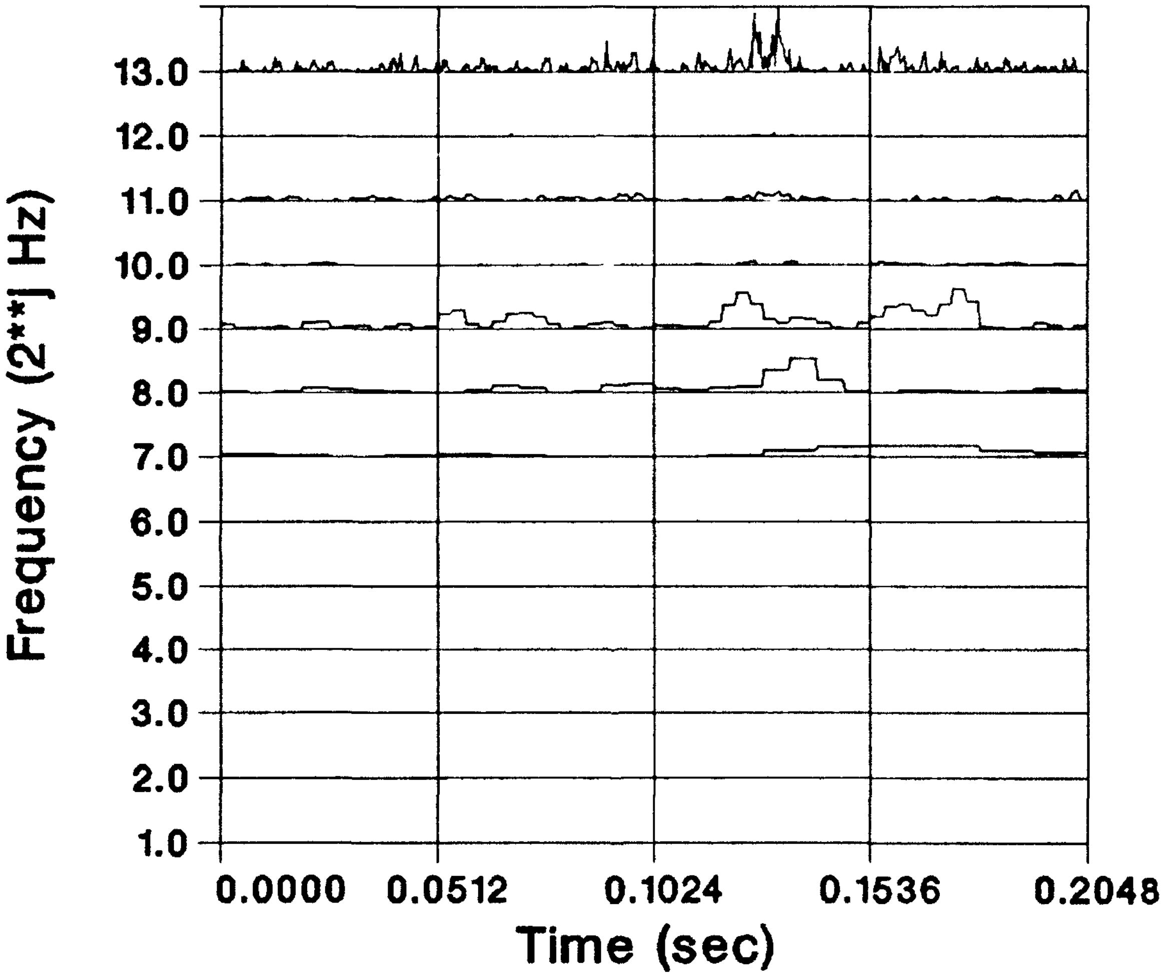
(b)

Figure 12. Time patterns of actual pump signal (a) without and (b) with abnormal condition.



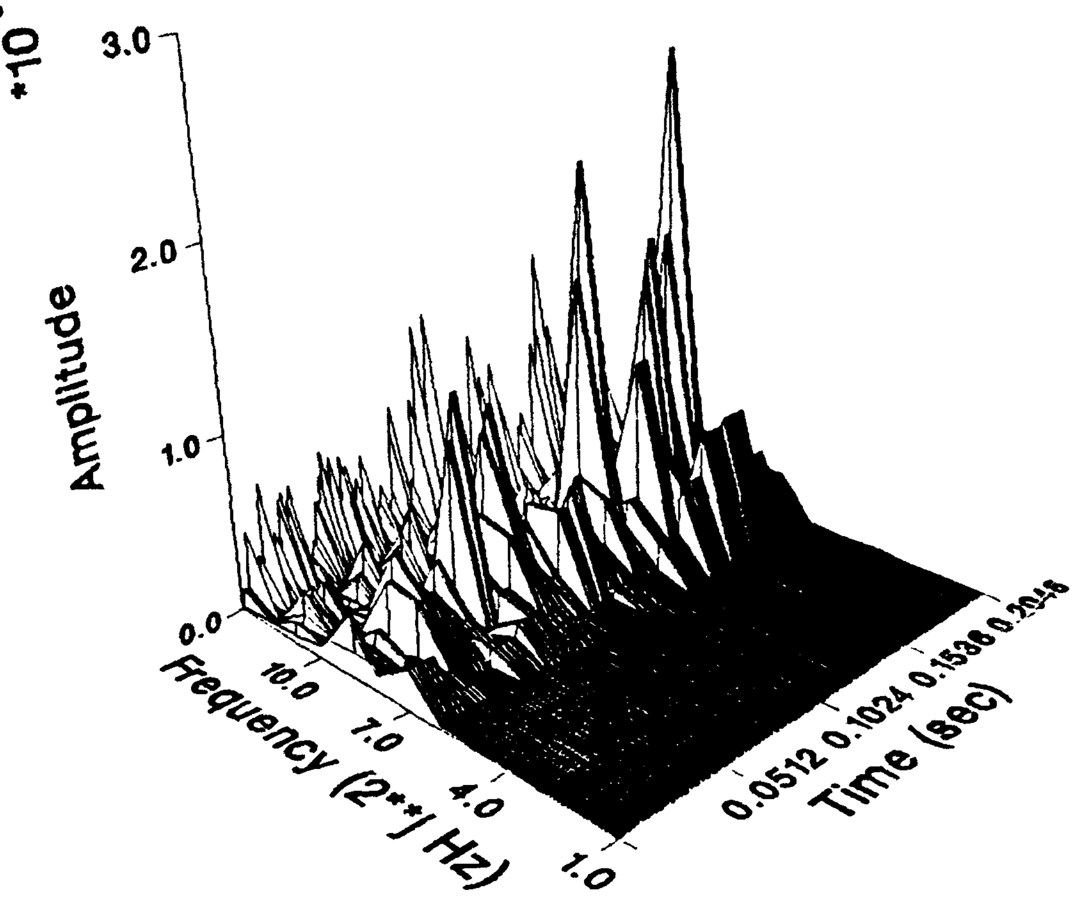
(a)

Fig.13 (continued)



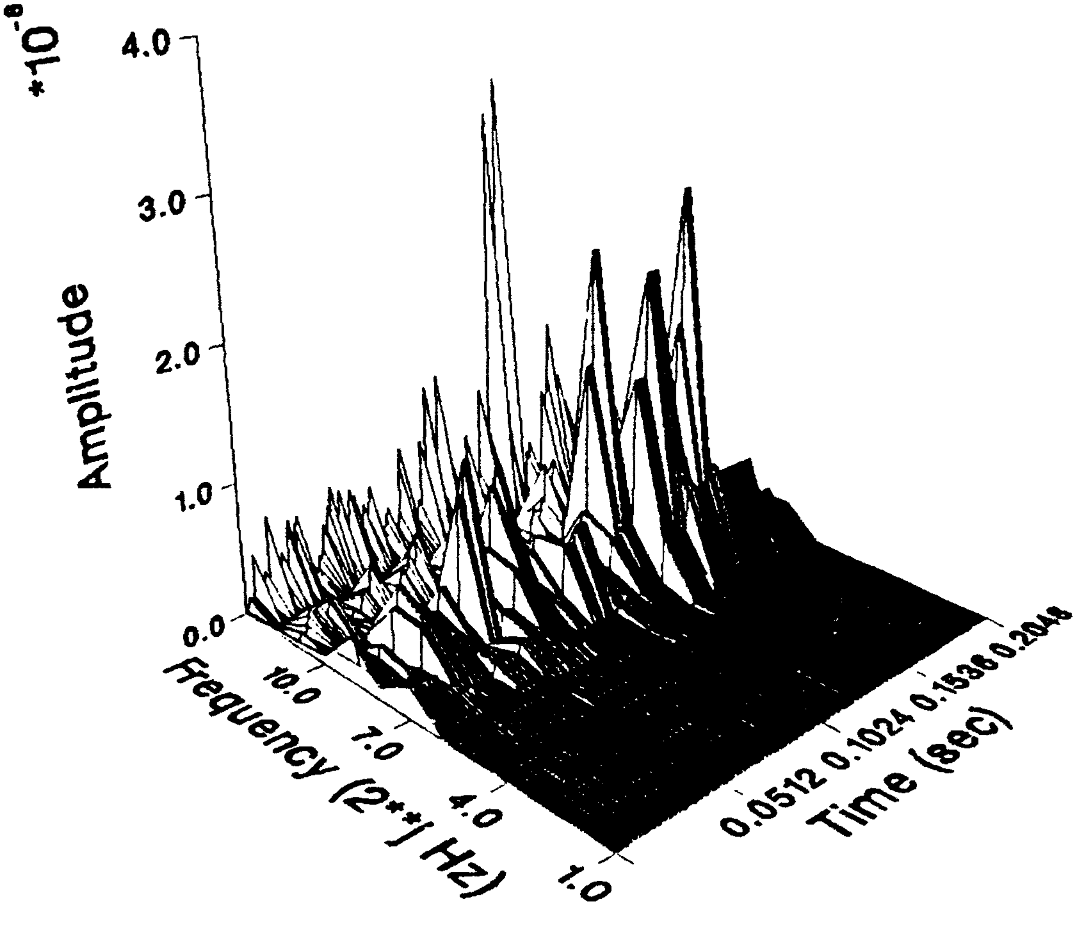
(b)

Figure 13. Time-frequency localization 2-D map of wavelet transform for Fig. 12(a) and (b), respectively (f_S = 10 kHz, N=2048).



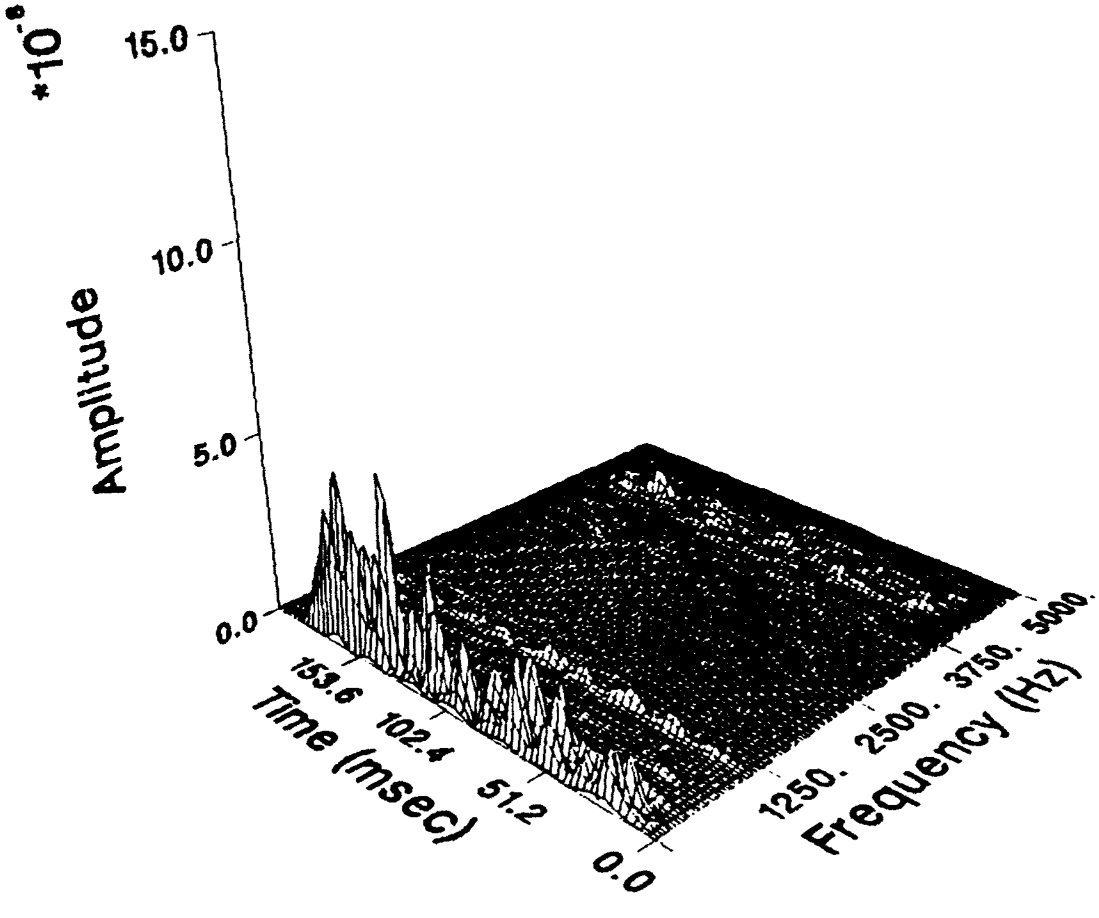
(a)

Fig.14 (continued)



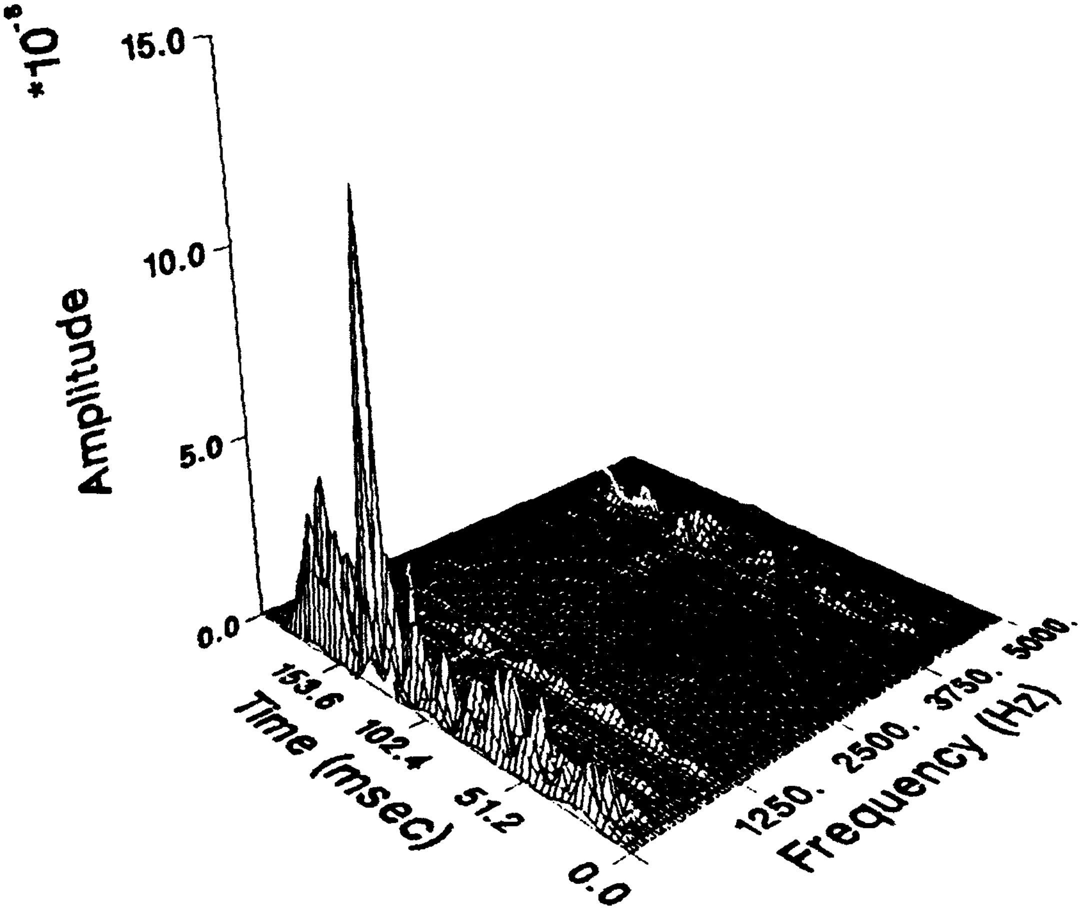
(b)

Figure 14. Time-frequency localization 3-D map of wavelet transform for Fig. 12(a) and (b), respectively (f_S=10 kHz, N=2048).



(a)

Fig.15 (continued)



(b)

Fig.13 Time-frequency localization of PWVD for Fig. 12(a) and (b), respectively (f_S = 10 kHz, N = 2048).

VI. CONCLUSIONS

The waveiet transform has been investigated and applied to analyzing sampled signal and the actual pump signal. The results of this research will be valuable asset for the analysis of vibration records and condition monitoring of machinery. The following conclusions can be drawn:

(1) The wavelet transform is ideally suited for portraying the wide-band transient or nonstationary vibration records in time-frequency domain.

(2) The wavelet transform has a great advantage to detect the small disturbance of the signal.

(3) The computation time of wavelet transform is very short in comparison with other time-frequency localization techniques.

(4) The modified Gaussian wavelet was well behavior and very effective to analyze the vibration records.

(5) The wavelet transform characterizes the time-frequency localization of the signal well and may be useful tool for the machinery condition monitoring.

(6) If the wavelet level, that is, frequency band, is selected at wavelet transform, its results will be a useful tool for the effective pattern recognition of machinery diagnosis and monitoring.

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APPENDIX. PROGRAM LIST

A. Program List of Wavelet Transform(FORTRAN 77)

```
c
      program wavelet
c
c -------------------------------------------------------------------
c
c                       WAVELET TRANSFORM
c
c                              by
c
c               Dept. of Mechanical Engineering
c                 Naval Postgraduate School
c
c -------------------------------------------------------------------
c
c     This program is a wavelet transform by using modified
c     Gaussian wavelet (octave sweep or 1/3 octave sweep).
c
c     Variables
c
c     n    = number of input data(n must be the power of 2.)
c     fs   = sampling frequency
c     dt   = sampling time(time interval)
c     bw   = cutoff frequency in highpass filter
c     a    = dilation
c     b    = translation
c     w    = coefficient of wavelet transform |w(a,b)|**2
c
c     Array
c
c     x(i) = input data set
```

```
c       x1(i) = filtered input data set
c       bk(i) = filter weight
c
c
        dimension x(8192),bk(4100),x1(8192)
        complex  ai,sum
        character*16 inname,outname
        character as
c
        ai=cmplx(0.,1.)
        pi=atan(1.)*4.
c
        print*,'=========================================='
        print*
        print*,'                    WAVELET TRANSFORM'
        print*
        print*,'=========================================='
        print*
        print*,'What is an input filename ?'
        read(*,10) inname
        print*,'What is an output filename ?'
        read(*,10) outname
10      format(a16)
c
c       select the sweep method
c
        print*,'What do you want  the sweep method ?'
        print*,' if 1/1 octave, input 1'
        print*,' if 1/3 octave, input 2'
        read(*,*) mswp

c
c       read the input data file
c
        call  indata(inname,x,fs,n,nn)
c
```

```
      dt=1./fs
c
      call smean(n,x)
c
      print*,'Do you want to apply highpass digital'
      print*,'filter to the original data ? (y/n)'
      read(*,20) as
20    format(a1)

      if (as.eq.'Y'.or.as.eq.'y') then
         print*,'Enter the cutoff frequency of'
         print*,'the digital highpass filter (in Hz)'
         read(*,*) bw
      endif
c
c     signal modifications
c
c     Application of highpass digital filter
c
      if (as.eq.'Y'.or.as.eq.'y') then
         mo=n/2

c     calculate the filter weighting

      call filter(mo,bw,dt,bk)

      write(*,*) 'finished filtering'

c     pass the highpass filter

      do 160 i=1,n
160   x1(i)=0.

      do 200 i=1,n
         do 170 k=-mo,mo
            j=k
```

```
             if (k.lt.0) then
                    j=k*(-1)
             endif
             j=j+1
             ll=i-k
             if (ll.lt.1) then
                    ll=ll+n
             elseif (ll.gt.n) then
                    ll=ll-n
             endif
             bb=-bk(j)
             if (k.eq.0) then
                    bb=1-bk(j)
             endif
             x1(i)=x1(i)+bb*x(ll)
170       continue
200    continue

       do 210 i=1,n
210       x(i)=x1(i)
       endif

       if (mswp.eq.1) then
          nk=nn
          fac=1.
       elseif (mswp.eq.2) then
          nk=3*nn
          fac=3.
       endif
       nr=nk-int(fac)+1

       open(7,file=outname,status='new')
c
c      calculate wavelet transform
c
       tini=0.
```

```
ttime=dt*n
f0=2.*pi
write(7,*) tini
write(7,*) ttime
write(7,*) nn,index

do 1000 i=1,nr
      fio=2**((nk-i+1)/fac)
      a=fio*dt
      nm=int(ttime/a)
      if (nm.eq.0) nm=1
      ddt=ttime/nm
      sr=sqrt(-alog(0.01)*2)*fio
      max=int(sr)
      if (max.ge.2*n) max=2*n

      do 900 j=1,nm

            sum=cmplx(0.,0.)
            b=ddt/2.+ddt*float(j-1)
            jk=int(b/dt)+1

            do 800 k=-max,max

                  t=k*dt/a
                  k1=jk+k
                  if (k1.lt.1) then
                        if (k1.lt.-n) then
                              k1=k1+2*n
                        else
                              k1=k1+n
                        endif
                  endif
                  if (k1.gt.n) then
                        if (k1.ge.2*n) then
                              k1=k1-2*n
```

```
                        else
                              k1=k1-n
                        endif
                  endif
                  sum=sum+x(k1)*cexp(-ai*f0*t)*exp(-t*t/2.)*dt

800           continue

              w=cabs(sum)/sqrt(a)
              w=abs(w)*abs(w)
              write(7,*) i,j,w

900       continue
1000   continue
       close(7)
       stop
       end
c
c
c
c      SUBROUTINES
c
c ---------------------------------------------------------------------------------
       subroutine indata(inname,x,fs,n,nn)

       dimension x(*)
       character*16 inname
c
       open(5,file=inname,status='old')
       read(5,*) fs,n
       do 100 j=1,n
          read(5,*) t,x(j)
100    continue
       close(5)
       ft=fs/2.
       do i=1,20
```

```
            ii=2**i
            di=float(ii)
            if (di.ge.ft) then
                  nn=i
                  go to 200
            endif
      end do
200   continue
      return
      end
c
c
      subroutine smean(n,x)
c
c     This subroutine calculates and removes the mean value of
c     the sampled data.
c
      dimension x(*)
      real meanv
c
      asum=0.
      do 100 i=1,n
            asum=asum+x(i)
100   continue
      meanv=asum/n
      do 200 i=1,n
            x(i)=x(i)-meanv
200   continue

      return
      end
c
c
      subroutine filter(mo,b,t,bk)
c
c     Routine generates FIR filter weights.
```

```
c       Method devised by Potter, Bickford and Glaze.
c       There are a total of 2M+1 weights...filter generates M+1.
c
c       --- variables ---
c       t = the sampling interval in second.
c       b = cutoff(half-power point) of the filter in Hz;
c             must be on the range from o to 1/2t.
c
c       Results are stored in bk
c
c  ---- Note; in the case of highpass filter, the value of
c             weight b0 must use 1-b0 instead of b0.
c
        dimension bk(*),d(3)
        data d0/0.35577019/,d(1)/0.2436983/,d(2)/0.07211497/,
       * d(3)/0.00630165/
        pi=atan(1.)*4.
        m=mo
c       first generate plain boxcar weights
        fact=2.*b*t
        bk(1)=fact
        fact=fact*pi
        do 5 i=1,m
        fi=i
5       bk(i+1)=sin(fact*fi)/(pi*fi)
c       trapezoidal weighting at end
        bk(m+1)=bk(m+1)/2.
c       Now apply the Potter p310 window
        sumg=bk(1)
        do 15 i=1,m
        sum=d0
        fact=pi*float(i)/float(m)
           do 10 k=1,3
10                sum=sum+2.*d(k)*cos(fact*float(k))
        bk(i+1)=bk(i+1)*sum
15      sumg=sumg+2.*bk(i+1)
```

```
      m1=m+1
      do 20 i=1,m1
20     bk(i)=bk(i)/sumg
       return
       end
```

B. Program List of wavelet transform(MATLAB)

```
%
%                Wavelet Transform
%
%                       by
%          Dept. of Mechanical Engineering
%              Naval Postgraduate School
%
%                Ver. 1.0      Aug. 5 1993
%
%                By using MATLAB Ver. 4.0
%
%     -- Variables
%          infile = input filename
%          fs     = sampling frequency
%          n      = the number of sampled dat
%                     ( n must be n-th power of 2.)
%          avg    = mean value of sampled data
%          fact   = the factor of sweep rate
%          x(i)   = the magnitude of signal
%          z(i,j) = the result of wavelet transform
%
     load c:\users\infile
     fs=infile(1,1);
     n=infile(1,2);
     dt=1/fs;
     x=infile(2:n+1,2);
     asum=0;
%
%    remove mean value
%
     for j1=1:n
          asum=asum+x(j1);
     end
```

```
avg=asum/n;
for i1=1:n
      x(i1)=x(i1)-avg;
end
%
%     Decide the sweep rate
%
fact=input('sweep method: 1/1 octave =1, 1/3 octave = 3 ');
%
ft=fs/2;
for i1=1:20
      ii=2^i1;
      if ii >= ft
         nn=i1;
         break
      end
end
if fact == 1
      nk=nn;
else
      nk=fact*nn;
end
nr=nk-fix(fact)+1;
%
%     calculate wavelet transform
%
tini=0;
ttime=n*dt;
f0=2*pi;
jm=fix(ttime/(2*dt));
wt=zeros(nr,jm);

for i1= 1 : nr
   io=2^((nk-i1+1)/fact);
   a=io*dt;
    nm=fix(ttime/a);
```

```
 if nm == 0
   nm = 1;
end
ddt=ttime/nm;
sr=sqrt(-log(0.01)*2)*io;
mx=fix(sr);
if mx >= (2*n)
   mx=2*n;
end

for j=1:nm
   sum=0+i*0;
   b=ddt/2+ddt*(j-1);
   jk=fix(b/dt)+1;
   for k=-mx:mx
      t=k*dt/a;
      k1=jk+k;
      if k1 < 1
         if k1 < -n
            k1=k1+2*n;
         else
            k1=k1+n;
         end
      end
      if k1 > n
         if k1 >= 2*n
            k1=k1-2*n;
         else
            k1=k1-n;
         end
      end
      x1=0.-i*f0*t;
      sum=sum+x(k1)*exp(x1)*exp(-t*t/2)*dt;
   end
   w=abs(sum)/sqrt(a);
   w=abs(w)*abs(w);
```

```
            wt(i1,j)=ww;
%
%       Plotting
%
        line=128;
        z=zeros(nr,line);
        for i1=1:nr
           io=2^((nk-i+1)/fact);
           a=io*dt;
           k=fix(ttime/a);
           if k == 0
             k=1;
           end
           if k <= line
             step=line/k;
             k2=0;
             for ii=1:k
                k1=k2+1;
                k2=fix(step*ii);
                st=k2-k1+1;
                if  st <= step
                    cf=step/st;
                else
                    cf=st/step;
                end

                for ij=k1:k2
                    z(i1,ij)=wt(i1,ii)/cf/st;
                end
             end
           else
             step=k/line;
             k2=0;
             for ii=1:line
                k1=k2+1;
                k2=fix(step*ii);
```

```
          st=k2-k1+1;
          ssum=0;
          for ij=k1:k2
              ssum=ssum+wt(i1,ij);
          end
          if st <= step
              ssum=ssum*step/st;
          else
              ssum=ssum*st/step;
          end
          z(i1,ii)=ssum;
      end
   end
end
xma=max(z);
xp=max(xma)*1.2;
dt1=ttime/(line-1);
for k=1:line
   xvalue=(k-1)*dt1;
   xx(k)=xvalue;
end
for k=1:nr
   yy(k)=k;
end

surf(xx,yy,z)
xlabel('Time (sec)')
ylabel('Frequency step ')
zlabel('Amplitude')
axis([tini ttime 1 nr 0 xp])
```

C. Plotting Program List (3 Dimension)

- FOTRAN 77 and CA-DISSPLA Graphic Package -

```
c
      program plot
c
c     This program uses the graphic package CA-DISSPLA to
c     plot the results of wavelet transform
c
c     tmax     = time record length
c     tint     = initial time
c     fmin     = start frequency step
c     fmax     = stop frequency step
c     nn       = the number of the frequency step
c     fac      = index of sweep method
c     fname    = input data filename generated by main source
c                program
c
      dimension rr(32768),wt(50,256)
      character*25 fname
c
      write(*,*) 'input file name ?'
      read(*,20) fname
20    format(a25)
c
c     data distribution or reduction for 3-D graphics
c
      line=256

      open(8,file=fname,status='old')

      read(8,*) tini
      read(8,*) tmax
      read(8,*) nn,n,fac
      nk=nn*int(fac)
      nr=nk-int(fac)+1
```

```
      ddt=tmax/n
      do 1000 i=1,nr
            fio=2.**((nk-i+1)/fac)
            a=fio*ddt

      k=int(tmax/a)
      if (k.eq.0) k=1
      if (k.le.line) then
            step=float(line)/float(k)
            k2=0
            do 950 ii=1,k
                 read(8,*) i1,j1,w
                 k1=k2+1
                 k2=int(step*ii)
                 st=float(k2-k1)+1.
                 if (st.le.step) then
                    coe=step/st
                 else
                    coe=st/step
                 endif
                 do 930 ij=k1,k2
                    wt(i,ij)=w/coe/st
930              continue
950         continue
      else
            step=float(k)/float(line)
            k2=0
            do 800 ii=1,line
                 k1=k2+1
                 k2=int(step*ii)
                 sum=0.
                 st=float(k2-k1)+1.
                 do 750 ij=k1,k2
                    read(8,*) i1,j1,w
                    sum=sum+w
750              continue
```

```
            if (st.le.step) then
               sum=sum*step/st
            else
               sum=sum*st/step
            endif
            wt(i,ii)=sum
800       continue
        endif
1000  continue

      close(8)

      smax=0.
      do 2000 i=1,nr
        do 2000 j=1,line
          if (smax.le.wt(i,j)) smax=wt(i,j)
2000  continue
      write(*,*)  'maximum=',smax
      write(*,*) 'input the maximum scale ?'
      read(*,*) fact

      do 120 i=1,nr
         do 100 j=1,line
          k=line*(i-1)+j
          rr(k)=wt(i,j)
100      continue
120   conunue

      dd=mod(nr,2)
      if (dd.eq.0) then
         nr=nr+1
         kk=(nr-1)*line+1
         kl=nr*line
         do 150 ip=kk,kl
150        rr(ip)=0.
      endif
```

```
c
c        plotting
c
         call pdev('ln03',ieer)
         call hwshd
         call swissm
         call shdchr(90.,1,0.002,1)
         call height(0.15)
         call physor(1.1,1.2)
         call area2d(5.5,6.75)
         call messag('WAVELET TRANSFORM $',100,1.1,8.2)
         call blsur
         call volm3d(8.,8.,9.)
         call x3name('Time (sec) $', 100)
         call y3name('Frequency step', 100)
         call z3name('Amplitude $', 100)
         call zaxang(90.)
         fmax=float(nr)
         fstep=(fmax-1)/4.
         call graf3d(tini,tmax/4.,tmax,1.,fstep,fmax,0.,'SCALE',fact)
         call surmat(rr,1,line,1,nr,1)
         call end3gr(0)
         call endpl(0)
         call donepl
         end
```

D. Plotting Program List (2 Dimension)

- FOTRAN 77 and CA-DISSPLA Graphic Package -

```
c
      program levelplot
c
c     This program uses the graphic package CA-DISSPLA to
c     plot the results of wavelet transform with respect to each level.
c
c     tmax      = time record length
c     tint      = initial time
c     fmin      = start frequency step
c     fmax      = stop frequency step
c     nn        = the number of the frequency step
c     fac       = index of sweep method
c     inname    = input data filename generated by main source
c                 program
c
      dimension x(512),y(512),wt(50,256)
      character*25 inname
c
      write(*,*) 'input file name ?'
      read(*,20) inname
20    format(a25)
c
c     data distribution or reduction for 2-D graphics
c
      line=512

      open(8,file=fname,status='old')

      read(8,*) tini
      read(8,*) tmax
      read(8,*) nn,n,fac
      nk=nn*int(fac)
      nr=nk-int(fac)+1
      ddt=tmax/n
```

```
      do 1000 i=1,nr
         fio=2.**((nk-i+1)/fac)
         a=fio*ddt
         k=int(tmax/a)
         if (k.eq.0) k=1
         if (k.le.line) then
            step=float(line)/float(k)
            k2=0
            do 950 ii=1,k
               read(8,*) i1,j1,w
               k1=k2+1
               k2=int(step*ii)
               st=float(k2-k1)+1.
               if (st.le.step) then
                  coe=step/st
               else
                  coe=st/step
               endif
               do 930 ij=k1,k2
                  wt(i,ij)=w/coe/st
930            continue
950         continue
         else
            step=float(k)/float(line)
            k2=0
            do 800 ii=1,line
               k1=k2+1
               k2=int(step*ii)
               sum=0.
               st=float(k2-k1)+1.
               do 750 ij=k1,k2
                  read(8,*) i1,j1,w
                  sum=sum+w
750            continue
               if (st.le.step) then
                  sum=sum*step/st
```

```
              else
                 sum=sum*st/step
              endif
              wt(i,ii)=sum
800         continue
          endif
1000    continue

        close(8)

        smax=0.
        do 2000 i=1,nr
           do 2000 j=1,line
              if (smax.le.wt(i,j)) smax=wt(i,j)
2000    continue
        write(*,*)  'maximum=',smax
        write(*,*) 'input the maximum scale ?'
        read(*,*) smax
c
c        normalizing
c
        do 2500 i=1,nr
           do 2500 j=i,line
2500          wt(i,j)=wt(i,j)/smax

c
c       plotting
c
        call pdev('ln03',ieer)
        call hwshd
        call swissm
        call shdchr(90.,1,0.002,1)
        call height(0.15)
         call page(8.5,11.)
        call physor(1.5,1.5)
        call area2d(4.8,4.7)
```

```
      call xname('Time (sec) $', 100)
      call yname('Frequency step', 100)
      call headin('WAVELET TRANSFORM $',100,1.1,1)
      call thkfrm(0.01)
      call yaxang(0.)
      call graf(tini,tmax/4.,tmax,1.,1.,nr+1)
      call grid(1,1)
      do 3000 i=1,nr
         do 2700 j=1,line
           x(j)=dt*float(j)
           y(j)=float(i)+wt(i,j)
2700     continue

      call curve(x,y,line,0)

3000  continue

      call endpl(0)
      call donepl
      stop
      end
```

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